

# LECTURE 1

In these lectures: modular semantics for DP and CP comparatives:

DP: Fred is (at least 10 cm) taller than Susan

CP: Fred is (at least 10 cm) taller than Susan is - / ever as -

Modular semantics:

-scalar types interpreted as rich scales.

-the meaning of scalar expressions stays as long as possible at scalar types and operations on scalar types.

-scalar meanings only become non-scalar through type driven type shifting.

Opposite view:

Doetjes (AC-paper) – van Rooy (vagueness paper) – Kamp 1975 – Klein 1980:

-Comparatives are derived through a semi-modal analysis from non-scalar adjective meanings.

-comparative meanings become only scalar when driven in context.

Two issues:

1. scalar versus non-scalar interpretations of adjectives:

$\lambda x. TALL(x,w)$  versus  $\lambda x. TALL_{cm}(x,w) >_{TALL} HIGH_{TALL,w}$

scalar effects with adjectives: modification with scalar operations:

*rather/very/very very... tall*

2. scalar versus modal definition of comparative.

*tall* versus *taller*

$\lambda x. TALL(x,w)$

$\lambda x. TALL_{cm}(x,w) >_{TALL} HIGH_{TALL,w} >_{TALL}$

## 1. The cross-linguistic argument (Kamp 1975):

cross-linguistically comparatives are morphologically derived from adjectives, the comparative is the more complex form.

hence: the semantics too should derive the comparative meaning from the adjective meaning:

semi-modal definition of comparative relation  
(Kamp, Klein)

Non-scalar adjective meaning

→ semi-modal definition of scalar operations (Klein)

## 2. The acquisition argument (e.g. van Rooy 2009):

Common acquisition wisdom: adjectives are acquired considerably before comparative and other scalar operations.

hence: the semantics too should derive non-scalar adjective meanings before scalar meanings.



**Kamp:** theory of vagueness with truth/falsity/undefinedness relative to precision states. The essential idea: the comparative has a **quasi-modal analysis**, defined in terms of the truth-falsity of the predication at **some** (possibly counterfactual) precision state.

Supervaluation theories of vagueness, Kamp 1975, 1975: vagueness as a modal notion: vagueness is *potential sharpness*.

Predicate *tall* is vague in state *s* iff within the set of ways in which the predicate can be made precise, the the line between the tall and the non-tall can still be drawn in different ways.

Set of *precision states* *S* and a partial order of precisification  $\leq$ , and a predicate like *tall* is assigned in a precision state *s* three subsets of the domain *D*:

-a **positive extension**:  $TALL_s^+$ , the individuals in *D* that are, according to *s*, definitely tall.

-a **negative extension**  $TALL_s^-$ : the individuals in *D* that are, according to *s*, definitely not-tall.

-a **gap**  $TALL_s^\perp$ , the individuals in *D* that are according to *s*, borderline cases.

Constraints on the structure make sure that precisification is monotonic and that vague predicates are potentially precise:

-**Partition**:  $TALL_s^+$ ,  $TALL_s^\perp$  and  $TALL_s^-$  partition the domain *D*.

-**Monotonicity**: if  $s_1 \leq s_2$ , then  $TALL_{s_1}^- \subseteq TALL_{s_2}^-$  and  $TALL_{s_1}^+ \subseteq TALL_{s_2}^+$ .

-**Precise states**: for every state *s* there is a state *t* such that  $s \leq t$  and for each predicate *P*:  $P_t^\perp = \emptyset$ .

Analysis of comparatives (mixture of Klein, Kamp, Landman 1992, etc.)

-Assume a notion of a degree predicate (see below)

Let TALL be a degree predicate.

$d_1 >_{TALL} d_2$  iff  $\exists s \in S: d_1 \in TALL_s^+$  and  $d_2 \notin TALL_s^+$  or  $d_s \in TALL_s^-$  and  $d_1 \notin TALL_s^-$

The relation that holds between Littlebro and Bigsis iff **there is a state *s* where** Littlebro is not tall and Bigsis isn't not tall, or Bigsis is tall and Littlebro isn't tall.

Modal relation (existential quantification over alternative precision states).

Presupposes a definition of degree predicate.

TALL is a degree predicate iff TALL satisfies the **degree predicate constraint**:

**Degree predicate constraint:**

Let  $\downarrow_s$  be the set of maximal chains in *S* going down from *s* (chains of vagueness increasing states, precision reduction states)

If  $d_1 >_{TALL} d_2$  then

$\forall s$  [ if  $d_2 \in TALL_s^+$  then  $\forall b \in \downarrow_s: \exists s' \in b: d_1 \in TALL_{s'}^+$  and  $d_2 \notin TALL_{s'}^+$  ] and  
 $\forall s$  [ if  $d_1 \in TALL_s^-$  then  $\forall b \in \downarrow_s: \exists s' \in b: d_2 \in TALL_{s'}^-$  and  $d_1 \notin TALL_{s'}^-$  ] and

For any individuals Littlebro and Bigsis:

If Bigsis  $>_{TALL}$  Littlebro then:

- any state where Littlebro is tall **can only be reached through** a state where Bigsis is tall, and Littlebro isn't yet and
- any state where Bigsis is not-tall **can only be reached through** a state where Littlebro is not-tall and Bigsis isn't yet.

### Observation:

The the modal semantics of the comparative is only correct if the predicate satisfies the degree predicate constraint.

**witness constraint:** difference in height (Bigsis is taller than Littlebro) is **witnessed** in the precisification structure of the predicates  $TALL_s^-$  and  $TALL_s^+$ .

**The witness constraint is (obviously) a scalar aspect of the meaning of tall. One can assume, in fact, that it is an aspect that only matures when comparative structures mature.**

### Problem 1: The intuitive basis.

Kamp: upward direction = deciding for more objects that they are in the positive and the negative extension. But: the extension is monotonic: you do not **revise** your criteria for membership, what is in the positive and negative extension stays in.

But then, is this predicate sharpening, and the inverse predicate blurring?

But intuitively, when you sharpen the criteria for membership, objects fall out of the positive extension, and when you blur, more gets in.

What is the relation between the formal relation on precision states and intuitive notions of defining and redefining standards of precision?

**Problem:** if it isn't clear what the order on precision states represents, then the degree predicates constraint is motivated by the definition of the comparative.

-If the degree predicate constraint is the claim that **if we restrict ourselves to monotonic increase operations**, the degree predicate denotations are ordered as the constraint says, then the definition **presupposes** a comparative ordering, rather than provide it.

i.e. we can define relation R if we **choose** exactly the notion of sharpening and blurring that gives us R.

-But if we don't have an independent criterion for why choose *that* notion of sharpening and blurring, it seems chosen mainly in order to get R.

-But then the definition doesn't define R, but presuppose R.

**-Problem 2: The same as for Klein (below).**

## Klein: McConnell-Ginet's idea:

- $\mu$  variable over adjective modifier functions,
- TALL an adjectival predicate
- op** = **orientation preserving**.

McConnell-Ginet:  $>_{TALL} = \lambda y \lambda x. \exists \mu [ \text{op}(\mu) \wedge ([\mu(TALL)](x)) \wedge \neg([\mu(TALL)](y)) ]$

$>_{TALL}$  = the relation that holds between Chuck and Barry iff there is an *orientation preserving function*  $\mu$  such that Barry is  $\mu$  tall and Chuck is not  $\mu$  tall.

**Idea:** Chuck and Barry are both very tall, but Barry is taller than Chuck. We can express this with orientation preserving functions *very very very* and *very very very very*: both Chuck and Barry are *very very very tall*, but only Barry is *very very very very tall*.

What does *orientation preserving* mean?

It is easy to express this notion in terms of the *scalar* meaning of the adjective.

-we associate with *short* and interval (0,low) and with *tall* an interval (high,  $\rightarrow$ ).

An orientation preserving function maps  $\mu$  maps (0,low) onto (0, $\mu$ (low)) and (high, $\rightarrow$ ) onto ( $\mu$ (high), $\rightarrow$ ).

-Intuitively  $\mu$  shifts the right-border of the interval for the adjectives that are oriented towards **low** and the left-border of the interval for adjectives that are oriented towards **high**.

Example: *very tall* shifts **high** to **very(high)** with **high** < **very(high)**  
shifts **low** to **very(low)** with **very(low)** < **low**

Easy to define scalarly (in fact, easiest to define in a modular semantics)

Klein must give a **non-scalar** semantics for orientation preserving modifier modifiers like *very*.

Klein 1980: semantics of *very* with help of the notion of **comparison set**.

Klein: comparison set is a contextual evaluation parameter for scalar adjectives:

adjectives can vary their extension with comparison sets.

-Out of the blue: *short* is evaluated relative to a contextually given comparison set:

*Fred is short* is true rel. **the population of Holland**, because Fred is considerably below the average height of Dutch males, or – for that matter – Dutch females.

*Fred is short* is false rel. **the population of Israel**, since he is above the average height of Israeli males.

(I accept that standards are set by a contextual property, but not a comparison set, see Kennedy's recent paper in L&P, also later in these lectures. i.e. I do not believe that *Chuck is short for a basketball player* means that *chuck is short within the set of basketball players*)

-a) Chuck may only be a candidate basketball player

-b) Intensional characterization:

Her voice is low for a singer of Mozart's Concert Aria Repertoire, like all other voices.)

### **The central semi-modal assumption: Wheeler 1972:**

Let  $s$  be a contextual comparison set.

*Chuck is very tall* is true rel  $s$  if *Chuck is tall* is true rel  $s \cap tall$

Semi-modal: the truth of *very tall*( $x$ ) relative to comparison set  $s$  is defined in terms of the truth of *tall*( $x$ ) relative to a different comparison set ( $s \cap tall$ ).

-Existential quantification over orientation preserving functions and resetting of comparison set comes down to existential quantification over alternative comparison sets (similar to Kamp).

As for Kamp, the comparison will only have the correct meaning if every comparative difference is **witnesses in some comparison set**:

Let  $TALL_s^+$  be the positive extension of TALL relative to comparison set  $s$ .

### **Witness assumption:**

If  $d_1 >_{TALL} d_2$  then  $TALL_{\{d_1, d_2\}}^+ = \{d_1\}$

if Bigsis is taller than Littlebro then Bigsis is tall rel. to comparison set  $\{Littlebr, Bigsis\}$  and Littlebro is not tall rel. to  $\{Littlebro, Bigsis\}$ .

The assumption looks well motivated when considering cases like the following:

(1) Karlsson holds up his hand with two clearly very big toffees. While both are big,

they are, nevertheless, clearly of different size. Karlsson says:

*Do you want the small one or the big one?* (cf. Lindgren 1959)

(2) ASSOCIATE: The Chrysler Bulding and the Empire State Building are for sale.

DONALD: *Buy the big one.*

(3) Two heavy suitcases, one even more more heavy than the other.

*"Out of the way, I'll take the heavy one upstairs first."*

However, these cases are **non-typical**.

(4) Albertine and Andree are the most lovely persons I know. Even so, Albertine is

just a tiny bit more lovely than Andree. According to Klein I must accept:  
*Of Albertine and Andree, Albertine is lovely and Andree is not.*

But I don't: both are lovely in **every** context that respects my standards for loveliness.

Possible reply: Think of comparison set {Albertine, Andree} as the comparison set in a context where you put a pistol on my chest and force me to chose one, if I think there is a difference.

- (5) a. Answer or die: only one of Albertine and Andree is lovely. Which one?  
b. Only one of these two beautiful ladies can be his LOVELY and he is going to choose now: Fred, which one is Lovely?

Problem: In this context, *lovely* does **not** have its normal (= non-competitive meaning).  
*Lovely shifts* to mean: *the more lovely one.*

More examples:

(6) Two plates of hot soup, one steaming, the other boiling. I ask:  
*Do you want the hot one?*

(7) Dante watches Brutus and Judas in hel. Virgil says, pointing at Judas:  
*Do you want to talk to the wicked one?*

(8) Sven Kramer, Lee Seung-hoon and Ivan Skobrev are at the podium at the  
5000 meters skating medal ceremony at the Olympics in Vancouver. The television announcer says:

*In the middle you see the fast one, on either sides are the not fast ones.*

(9) Einstein and Feynman apply for a physics position. Search committee member:

*Let's take the clever one.*

(10) Two mixings of a heavy metal song:

*"I think we should put the loud mixing on the album."*

What about the exceptional cases?

-Donald has a linguistic handicap: in any domain only the biggest element counts as big.  
-Cf the losing skater: "It just didn't go today, I was slow." = Redefine *slow* as *anything that doesn't get a medal.*

Proposed analysis for the other cases (the opposite of Klein's assumption):

In exceptional cases, the meaning of *be small* and *be big* can be **stretched** to a comparative meaning: *be among the smaller ones* and *be among the bigger ones*:

(11) a. Do you want the small one or the big one?

b. Do you want the smal(*ler*) one (*of the two*) or the big(*ger*) one (*of the two*).

If so, you cannot use the exceptional interpretations in (11a) to motivate the semantics of *very* and hence the semantics of the comparative fails, because the exceptional interpretations themselves are defined in terms of the semantics of the comparative. Most scalar adjectives do not naturally allow for comparative reinterpretation. But all allow comparatives and modifiers like *very*.

**Moral:** Klein defines the meaning of the comparative in terms of the meanings that the parts have **and meanings that the parts don't have (i.e. comparative adjective meanings)**.

That is not Frege's principle.

With this the non-scalar semi-modal interpretation of *very* fails, and with it the reduction of the comparative to the positive. The same for other orientation preserving modifiers. Kamp's reduction fails for similar arguments: Kamp tells me that I must allow for Andree to be moved out of the positive extension of *lovely* before Albertine is, and this too, I refuse to do as long as *lovely* means what it does.

## RICH SCALES

### Modularity:

-from scale, domain of degrees to domain of individuals by type shifting with a measure function.  
-then the measure function must be recoverable from the degrees in the scale.

Scalar domain:  $\langle r, u, M \rangle$ :  $r \in \mathbf{R}$ :  $u$  is a unit and  $M$  a measure

Measure function  $M_u$  is recoverable.

**Rich scales:** real numbers, two way infinite (without  $-\infty, +\infty$ ), dense, linearly ordered, with infimums and supremums and arithmetics.

**van Rooy 2009:** weaker scales.

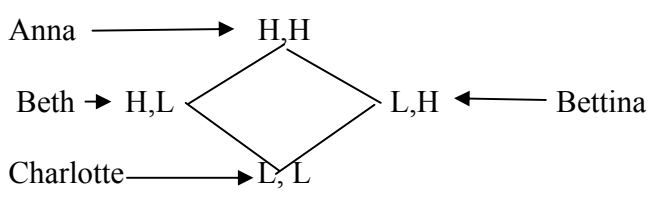
van Rooy 2009:

"Let us assume, for the sake of argument, that there are only two properties/dimensions associated with being clever: an ability to manipulate numbers, and an ability to manipulate people. Let us say that John is cleverer than Mary iff John is better both in manipulating numbers and in manipulating people. But now consider Sue. Sue is worse than John in manipulating numbers but better in manipulating people. Thus, neither John is cleverer than Sue, nor Sue is cleverer than John. For the *cleverer than*-relation still being almost connected, it has to be the case that Sue is cleverer than Mary. But it is well possible that although Sue is better at manipulating people than Mary (and John), Mary is better than Sue in manipulating numbers. Thus, if one doesn't fix a particular dimension, one cannot claim that *cleverer than* denotes a relation that is almost connected." van Rooy, *Vagueness and Linguistics* p. 25.



(If we assume a scale ordered by a linear order  $<$ , and a function  $\mu$  mapping individuals to degrees on the scale, then the induced relation between individuals  $\lambda y \lambda x. \mu(x) < \mu(y)$  is almost connected (it becomes a linear order by taking equivalence classes under the relation  $\lambda y \lambda x. \mu(x) = \mu(y)$ )).

Van Rooy :



We measure cleverness along two dimensions,  
 -H, H is more clever than anybody else  
 -L,L is less clever than anybody else  
 -H,L versus L,H?

van Rooy: incommensurable

I say: equal at the current state of calculating complex values.

For clarity replace *being more clever than* by a contextually neutral relation: *being more suited than* in a job search.

-We are looking for an administrator who is good at crunching numbers and at manipulating people.

Anna and Charlotte are not in the running.

Who is more suited; Beth or Bettina?

Why not say: they are equally suited (in different ways)?

equally suited = equally suited **given the** lack, in context, of a weighing procedure for complex suitedness. (Contextually **non-monotonic** notion).

This is possibly still a matter of esthetic choice, but there is a real problem.

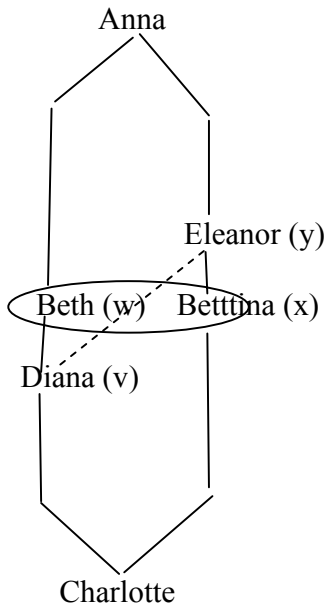
Van Rooy proposes weaker scalar structures, but not too weak, so as not to make defining measures possible:

**Semi orders** (e.g Luce 1956) irreflexive relations R such that:

Interval order:  $R(x,y) \wedge R(v,w) \wedge \neg R(x,w) \rightarrow R(v,y)$

Semi-transitive:  $R(x,y) \wedge R(y,z) \wedge \neg R(x,v) \rightarrow R(v,z)$

Problem: if van Rooy's objection to linear orders above is valid, it is valid against his own structures as well (and measuring is just not possible):



The same situation as before, but we add Eleanor and Diana.

-Eleanor's score on the first criterion is the same as Bettina's, worrisome.

But her score on the second criterion is significantly better than Bettina.

-Beth and Diana score the same on the second criterion, but Beth is significantly better on the first.

Nevertheless, these scores are in the same ballpark.

van Rooy's structures tell us: Eleanor is more suited than Diana

Me: I don't see that is natural given his own argument.

If we insist that Beth and Bettina are incomparable, it is more natural to regard Diana and Eleanor as incomparable as well.

-I am not convinced by van Rooy's argument concerning dimensions,

-Classical view on scales:

scales are linear orders with the structure corresponding to the reals.

Dimensional comparatives like *be more clever than* correspond in context to a multiplicity of scales:

-in context there may be available a scale for each dimension, scales for union dimensions (underspecified between two dimensions), intersection dimensions (calibrating the values along two dimensions), etc.

This is not something special, since, in the semantics a scale is assigned relative to a contextually specified property in the first place which intuitively is meant to constrain how the measure function calculates its values.

## MEASURE AND UNIT DETERMINE MEASURE FUNCTION UP TO EXPRESSIBLE EQUIVALENCE

The measure function is sensitive to contextual property  $k$ .

-Scales like height-in-cm where the **numerical values** are linguistically accessible, because the language contains unit-denoting expressions, which combine with number expressions: (*more than three*) **centimeters** (*taller*).

-Scales like *loveliness-in-heartbeats*, where the numerical values are not linguistically accessible (no unit-denoting expressions combining with number expressions): here the actual numerical values assigned by the measure function are not important: **tongue-in-cheek degrees**.

Nevertheless, tongue-in-cheek domains provide an argument for rich scales, because the proportional scalar relations **are** important in these domains, and they require domains to which arithmetics applies..

### **Multiplication:**

Even in scalar domains of tongue-in-cheek degrees multiplication expressions are possible with "very round" numbers:

- (1) a.  $\surd$  Albertine is *ten times/a hundred times more* lovely than Emma.  
b. #Albertine is *7.3 times more* lovely than Emma.

I see no reason to assume that these expressions do not have their normal meaning in these domains. Thus, if we make the assumption that for reasonably sized amounts of loveliness, twice an amount is a lot, the following inference, showing addition and multiplication, should be valid:

- (2) a. Mary is more lovely than Albertine and Elisabeth Bennett taken together.  
b. Sarah is just a bit more lovely than Albertine, and just a bit more lovely than Elisabeth Bennett.  
c. Mary is quite a bit more lovely than Sarah.
- (3) a. Mary is ten times more lovely than Albertine.  
b. Sarah is a hundred times more lovely than Mary.  
c. Sarah is a thousand times more lovely than Albertine.

We capture this by in general allowing the measure function to vary with respect to the context, but requiring it to preserve expressible properties:

**Constraint:** for measure  $M$ , unit  $u$  and contextual properties  $k_1$  and  $k_2$  and language  $L$ :  
 measure functions  $M_{u,k_1}$  and  $M_{u,k_2}$  are **expressibly equivalent relative to  $L$**   
 if

- $M_{u,k_1}$  and  $M_{u,k_2}$  are defined for the same inputs
- preserve the induced order on the input
- preserve induced supremums and infimums
- preserve special elements if the language expresses them  
 (like a 0 for height or loveliness)
- preserve those arithmetic properties that the language expresses.

**Example 1:** gauged scales like height-in-cm.

A measure function  $M_u$  is **gauged** iff for all  $k_1, k_2$ :  
 for all  $x, w$ :  $M_{u,k_1}(x, w) = M_{u,k_2}(x, w)$

For Height English allows full arithmetic access to the language:

English has unit expressions *centimeters, inches, ...* which combine with numeral expressions, *three centimeters*, and these expressions can be used as differentials in scalar expressions; *three centimeters taller, ...*

-The relation between a degree assigned by the measure function and degree 0 is expressible in the language, which means that the measure height-in-cm is forced to be gauged in English.

**Example 2:** Loveliness is a tongue-in-cheek measure. Numerical differentials are (out of the blue) not available.

-We find basic addition through sums.

-We find multiplication with rhetoric round numbers.

-the actual degree of loveliness assigned to Mary and Jane rel  $k_1$  to  $k_2$  may vary (since they are tongue-in-cheek-degrees), but if Mary is ten times as lovely as Jane in  $s_{k_1}$ , she is ten times as lovely as Jane in  $s_{k_2}$ , because the relation *ten times as lovely* is preserved:

$$\lambda y \lambda x \lambda s. L_{p,k}(x, s) = 10 \times L_{p,k}(y, s)$$

-Note: while the scale is unbounded in both directions, the measure function range need not be unbounded at all. For instance: both loveliness and height have expressible null-points.

**In sum:**

-the structures are classical, in that I assume issues concerning different degrees of precision, dimensional aspects (like the different senses of *clever*), and issues of non-linearity relating to that (see van Rooy 2009) to be fixed in the background context (the model).

-I do incorporate one aspect of measure theory (Suppes et. at., van Rooy): the idea that preservation constraints determine the difference between extensional scales and tongue-in-cheek scales.

-But I assume that the actual preservations required are not fixed by mathematics or logic, but are language dependent and even contextually manipulable, even fraudulently:

(4) Fraudulent Advertisement:

"When we did the measurements, Mrs. X was *37 lipels fatter than* was healthy and esthetic for her age and body frame. After our treatment *all 37 lipels* were gone."

# LECTURE 2

Let  $k$  be a **comparison property**, a property of individuals.

A **scale for  $k$**  is a structure  $s_k$  with the following ingredients:

1. **scale measure:**  $s_k^M$  is a measure  $M$  [Height, Age, ...]
2. **scale unit:**  $s_k^u$  is a unit  $u$ , appropriate for  $M$ . [cm, inch, ...]
3. **scalar domain:**  $s_k^\Delta$  is  $\Delta_{M,u} = \{ \langle r, u, M \rangle : r \in \mathbf{R} \}$  [triples based on the reals]

**Mnemonic superscripts** denote the relevant element of a tuple:

example:  $\langle 29, \text{cm}, H \rangle^r = 29$

## SCALAR DIRECTIONAL NOTIONS

4. **scalar direction:**  $s_k^> \in \{ >_M, <_M \}$  where:

$$>_M = \{ \langle \delta_1, \delta_2 \rangle : \delta_1, \delta_2 \in \Delta_{M,u} \text{ and } \delta_1^r > \delta_2^r \}$$

$$<_M = \{ \langle \delta_1, \delta_2 \rangle : \delta_1, \delta_2 \in \Delta_{M,u} \text{ and } \delta_1^r < \delta_2^r \}$$

5. **scalar supremums:** if  $s_k^> = >_M$ , then  $s_k^u = \sqcup_{>_M}$   
 if  $s_k^> = <_M$ , then  $s_k^u = \sqcup_{<_M}$  where:

$$\sqcup_{>_M}(\Delta) = \langle \sqcup_{>}(\Delta^r), u, M \rangle \quad (= \langle \sqcap_{<_R}(\Delta^r), u, M \rangle)$$

$$\sqcup_{<_M}(\Delta) = \langle \sqcup_{<}(\Delta^r), u, M \rangle \quad (= \langle \sqcup_{<_R}(\Delta^r), u, M \rangle)$$

Supremum and Infimum on  $\mathbf{R}$ :

Let  $X \subseteq \mathbf{R}$  Let  $X_{<}^u = \{ y \in \mathbf{R} : \forall x \in X: x \leq y \}$ ,  
 the set of all numbers at least as big as every number in  $X$

$$\sqcup_{<}(X) = \begin{cases} \text{the } <\text{-minimum of } X_{<}^u, \text{ if } X_{<}^u \text{ has a } <\text{-minimum} \\ \text{undefined otherwise} \end{cases}$$

Let  $X_{<}^\square = \{ y \in \mathbf{R} : \forall x \in X: y \leq x \}$   
 the set of all numbers at least as small as every number in  $X$

$$\sqcap_{<}(X) = \begin{cases} \text{the } <\text{-maximum of } X_{<}^\square, \text{ if } X_{<}^\square \text{ has a } <\text{-maximum} \\ \text{undefined otherwise} \end{cases}$$

6. **scalar subtraction** if  $s_k^> = >_M$ , then  $s_k^- = \rightarrow_M$   
 if  $s_k^> = <_M$ , then  $s_k^- = \rightarrow_{<_M}$  where:

$$\rightarrow_M = \lambda \delta_2 \lambda \delta_1. \langle \delta_1^r - \delta_2^r, u, M \rangle \quad \text{subtraction}$$

$$\rightarrow_{<_M} = \lambda \delta_2 \lambda \delta_1. \langle \delta_2^r - \delta_1^r, u, M \rangle \quad \text{converse of subtraction}$$

We take notions of  $s_k^<$ ,  $s_k^\square$ ,  $s_k^+$  to be defined accordingly.

### 7. scalar high and low intervals

1.  $s_k^{\text{HIGH}}, s_k^{\text{LOW}} \in s_k^{\Delta}$  with  $s_k^{\text{HIGH}} > s_k^{\text{LOW}}$
2.  $s_k^{\uparrow}, s_k^{\downarrow}$  are intervals in  $s_k^{\Delta}$  with  $s_k^{\sqcup}(s_k^{\uparrow}) = s_k^{\text{HIGH}}$  and  $s_k^{\sqcap}(s_k^{\downarrow}) = s_k^{\text{LOW}}$
3. if  $k$  is **unbounded**:
  - $s_k^{\uparrow} = (s_k^{\text{HIGH}}, \rightarrow) = \{\delta \in s_k^{\Delta} : \delta > s_k^{\text{HIGH}}\}$
  - $s_k^{\downarrow} = (\leftarrow, s_k^{\text{LOW}}) = \{\delta \in s_k^{\Delta} : (s_k^{\text{LOW}} > \delta)\}$

Examples: *hot* and *warm* are  $>$ -oriented, right-unbounded

-if you're hot, you're warm

-if you are hot and something is hotter than you it has a higher temperature

-if you're hot, and something has a higher temperature, it is hot

*cold, cool* are  $<$ -oriented, and left unbounded

-if you're cold, you're cool

-if you are cold and something is colder than you it has a lower temperature

-if you're cold and something has a lower temperature it is cold

*tepid* is  $<$ -oriented but bounded

-if you're tepid, you're not tepid

-if you are tepid and something is more tepid than you it has a lower temperature

-if you are tepid and something has a lower temperature it is not necessarily tepid

### 8. scalar measure function:

$s_k^{\mu} = M_{u,k}$  where:

$$M_{u,k}: D \times W \rightarrow s_k^{\Delta}$$

**Constraint:**

$$\forall w \in W: k(w) = \lambda x. M_{u,k}(x,w) \in s_k^{\uparrow}$$

Out of the blue, the comparison property for the height scale may be the property TALL, i.e. we deal with scale  $_{\text{TALL}}$ , with  $_{\text{TALL}}^M = H$ , and  $_{\text{TALL}}^u = \text{cm}$  (say).

The constraint makes the connection between the non-scalar adjective meaning TALL and the scales meaning:

$$\forall w \in W: \text{TALL}(w) = \lambda x. H_{\text{cm},k}(x,w) \in _{\text{TALL}}^{\uparrow}$$

### COMPARISON PROPERTIES:

We take  $k$  to be a comparison property,  $\sqcap$  intensional conjunction, and  $0$  the null-property such that  $0 \sqcap P = P$ ,  $P \sqcap 0 = P$ , etc.

When  $\text{TALL} \sqcap k$  is the scalar comparison property, we think of  $k$  as a contextual restriction on TALL, restricting the extension of TALL to a contextual subset.

Note that the comparison property plays a similar contextual role to Klein's comparison set, but without the extensional connotations that Klein's sets have.

**Example:**

- (1) a. Lowly is tall.
- b. Giraffa is short.

-Context  $k$  chooses an appropriate default unit (cm)  
 -and determines a relevant property (*short for a worm/giraffe*) :  
 $SHORT \sqcap WORM \sqcap k / SHORT \sqcap GIRAFFE \sqcap k$ .

We get for (1a) and (1b):

- (1) a.  $H_{TALL \sqcap WORM \sqcap k, cm}(Lowly, w) \in S_{TALL \sqcap WORM \sqcap k}^{\uparrow}$
- b.  $H_{TALL \sqcap GIRAFFE \sqcap k, cm}(Giraffa, w) \in S_{TALL \sqcap GIRAFFE \sqcap k}^{\uparrow}$

The properties  $TALL \sqcap WORM \sqcap k$  and  $TALL \sqcap GIRAFFE \sqcap k$  determine scales  
 $S_{TALL \sqcap WORM \sqcap k}$  and  $S_{TALL \sqcap GIRAFFE \sqcap k}$ .

In this example these scales are **identical height-scales except in two respects:**

- 1.  $H_{TALL \sqcap WORM \sqcap k, cm}$  and  $H_{TALL \sqcap GIRAFFE \sqcap k, cm}$  differ from  $H_{TALL, cm}$  in that they may have **sortal restrictions:**

We may plausibly assume that  $H_{TALL \sqcap WORM \sqcap k, cm}$  is undefined for giraffes, and  
 $H_{TALL \sqcap GIRAFFE \sqcap k, cm}$  is undefined for a work.

- Cf.: (3) a. Hank (who is a basketball player) is short for a basketball player.  
 b. Buck (who wants to be a basketball player) is a bit short for a basketball player.  
 c.# Tiny Tim(who still has a trauma about basketball from school) is short for a basketball player.

The measure function would be defined for basketball players and candidate-basketball players, but not for Tiny Tim's who aren't in the ball park.

(Note: this precisely goes against Klein's analysis, Klein doesn't have candidates in the comparison set.)

- 2.  $S_{TALL \sqcap WORM \sqcap k}^{HIGH} < S_{TALL}^{HIGH} < S_{TALL \sqcap GIRAFFE \sqcap k}^{HIGH}$ .
- 2.  $S_{TALL \sqcap WORM \sqcap k}^{LOW} < S_{TALL}^{LOW} < S_{TALL \sqcap GIRAFFE \sqcap k}^{LOW}$ .

The high and low values on the scale, and consequently, the high and low intervals on the scale depend on the contextual properties:

what is tall for a worm is much lower than what is tall out of the blue and the latter again is much lower than what is tall for a giraffe.



## 9. Converses.

The converse scale of  $s_k$ ,  $(s_k)^c$  is the scale which is identical to  $s_k$  except that where  $s_k$  has orientation  $>_M$  ( $<_M$ ),  $(s_k)^c$  has orientation  $<_M$  ( $>_M$ ), with all dependent notions defined accordingly.

We define, accordingly converses for all relevant notions:

1.  $(s_k^M)^c = s_k^M$ ,  $(s_k^M)^u = s_k^u$ ,  $(s_k^\Delta)^c = s_k^\Delta$ ,  $(s_k^\mu)^c = M_{u,k}$ .
2.  $(s_k^{>})^c = s_k^{<}$
3.  $(s_k^\sqcup)^c = s_k^\sqcap$
4.  $(s_k^-)^c = -<_M$  if  $s_k^- = ->_M$   
 $(s_k^-)^c = ->_M$  if  $s_k^- = -<_M$
7.  $(s_k^H)^c = s_k^{LOW}$ ,  $(s_k^{LOW})^c = s_k^{HIGH}$   
 $(s_k^\uparrow)^c = s_k^\downarrow$ ,  $(s_k^\downarrow)^c = s_k^\uparrow$

### Constraint:

For comparison property  $k$ , we assume there is a unique comparison property  $k^c$  such that for every appropriate scale  $s_k$ :  $s(k^c) = (s_k)^c$

Example: if the semantics stipulates:  $COLD = HOT^c$

Then the constraints stipulate:  $HOT(w) = \lambda x. T_{\circ, HOT}(x, w) \in SHOT^\uparrow$   
 $COLD(w) = \lambda x. T_{\circ, HOT}(x, w)^r \in SHOT^\downarrow$

For clarity we will write  $s_{M,u,k}$  for scale  $s_k$  with  $s_k^M = M$  and  $s_k^u = u$

-We will make assumptions like the following:

$(TALL \sqcap GIRAFFE \sqcap k) = TALL^c \sqcap GIRAFFE \sqcap k$

-We will make assumptions like:

$WARM \sqsubseteq HOT$

-the scales of WARM and HOT are identical temperature -scales, except that:

$$s_{WARM, \circ}^{HIGH} < s_{HOT, \circ}^{HIGH}$$

-Special constraint on *tepid*:

for all  $d \in D$  for all  $w \in W$ :

if  $TEMP_{\circ, TEPID}(x, w) \notin s_{TEPID}^\uparrow$ , then  $TEMP_{\circ, TEPID}(x, w) = \perp$

(1) The soup is more tepid than the bathwater.

(1) is only defined, if the soup and the bath water are tepid.

We don't have a similar constraint for most adjectives:

*A is hotter than B* doesn't mean that A and B are hot,

even : *A is redder than B* may not be true for A and B that are distinctly not red, but can be true for A and B that are not red, but both have some reddish aspects.

**Idea:**

*tall* is ambiguous:

*tall* denotes  $S_{TALL}$

*tall* denotes  $S_{TALL}^M (= H)$

*short* is not ambiguous:

*short* denotes  $S_{TALL}^c$

Comparative: *adjective* + er is built from adjective meaning:

$S_{ADJECTIVE}$

three centimeters *adjective* is built from adjective meaning:  $S_{ADJECTIVE}^M$

Prediction: ✓ *tall-er*

✓ *short-er*

✓ three centimeters *tall* #three centimeters *short*

# TYPESHIFTING SEMANTICS

## The Principle BPR: (Bach, Partee, Rooth):

Interpret everything as low as you can, but not so low that you will regret it later.

## Modular semantics for degree phrases:

-*three* denotes 3.

-Keep the denotations of degree expressions at the level of degrees, predicates of degrees, etc. for as long as you can.

**Example:** scalar semantics of *rather*, *very* is straightforward:

Degree interpretation of *tall*:  $\lambda\delta.\delta >_H S_H^{HIGH}$

*Very*:  $S_{H,k} \rightarrow S_{H,very(k)}$ , where  $S_{H,very(k)}^{HIGH} >_H S_H^{HIGH}$

This means that the scalar meaning of *tall* must shift, later in the derivation.

PREDICATE OF DEGREES + MEASURE FUNCTION  $\rightarrow$

PREDICATE OF INDIVIDUALS

### COMPOSE WITH MEASURE FUNCTION

$$\alpha \quad \rightarrow \quad \alpha \circ M \quad (\text{where } \alpha \circ M = \lambda x. \alpha(M(x)))$$

$$\langle \delta, t \rangle \quad \rightarrow \quad \langle e, t \rangle$$

$$\lambda\delta. \delta >_H \langle 3, \text{cm}, H \rangle \circ H_{\text{cm},k} = \lambda x. H_{\text{cm},k}(x, w) >_H \langle 3, \text{cm}, H \rangle$$

The set of heights bigger than three cm      The set of individuals with height bigger than three cm

$$\lambda\delta. \delta >_H \langle 3, \text{cm}, H \rangle \circ H_{\text{cm},k}$$

### Generalized function composition

$$f \circ g = \lambda x_n \dots \lambda x_1. f(g(x_1, \dots, x_n))$$

(Bring function *g* down to the input type for *f*, by applying it to variables, apply *f* to the result, and abstract over all the variables used.)

1. Bring  $H_{\text{cm},k}$  down to the input type for  $\lambda\delta. \delta >_H \langle 3, \text{cm}, H \rangle$  by applying to variables:

$$H_{\text{cm},k}(x, w)$$

2. Apply  $\lambda\delta. \delta >_H \langle 3, \text{cm}, H \rangle$  to this:

$$\lambda\delta. \delta >_H \langle 3, \text{cm}, H \rangle (H_{\text{cm},k}(x, w))$$

=

$$H_{\text{cm},k}(x, w) >_H \langle 3, \text{cm}, H \rangle$$

3. Abstract over the variables applied to earlier:

$$\lambda w \lambda x. H_{\text{cm},k}(x, w) >_H \langle 3, \text{cm}, H \rangle$$

4. The world parameter is fixed in context:

$$\lambda x. H_{\text{cm},k}(x, w) >_H \langle 3, \text{cm}, H \rangle$$

Equivalently:

$$\lambda x. H_{\text{cm},k}(x, w)^f > 3$$

Similarly:  $\langle \delta, \langle \delta, t \rangle \rangle$  into  $\langle x, \langle \delta, t \rangle \rangle$ .

**-Modularity:** -scalar operations take place at the level of scales of degrees, degrees, predicates of degrees, relations between degrees.

-Since degrees are triples,  $\langle r, u, M \rangle$ , you can reconstruct the measure function up to contextual parameters from a set of degrees. This allows shifting.

### SEMANTIC ASSUMPTIONS IN MORE DETAIL:

1. *tall* has an interpretation as **measure H**, *short* does not.

2. *tall* and *short* have interpretations as **scales**.

(more precisely, functions from units to scales).

Let  $s_{k,u}$  be a scale  $s_k$  with  $s_k^u = u$

(out of the blue:

*tall* denotes the **basic scale** of Height:  $\lambda u. s_{TALL,u}$

*short* denotes the **converse scale** of Height:  $\lambda u. s_{TALL,u}^c$

3. The meaning of adjectives *tall/short* as scalar predicates and comparatives *tall/short* are derived from their **scalar** meanings.

4. *more* denotes the **difference function** (subtraction)

On numbers:

$$more = \lambda m \lambda n. n - m$$

maps two numbers  $n$  and  $m$  onto **what  $n$  is more than  $m$**

On scales:

$$more = \lambda s. s^-$$

maps a scale onto **its** difference function

*less* denotes the **converse of the difference function**

On numbers:

$$less = \lambda m \lambda n. m - n$$

maps two numbers  $n$  and  $m$  onto **what  $n$  is less than  $m$**

On scales:

$$less = \lambda s. s^-$$

maps a scale onto the converse of its difference function.



**STEP 1:**

-PRED<sub>num</sub>: null-predicate of numbers  $\emptyset_{a \text{ bit}}$ , with semantic meaning *a bit* (or *some*)

$$\emptyset_{a \text{ bit}} \rightarrow \lambda r. r >_{\mathbf{R}} 0 \quad (\text{the set of positive real numbers})$$

-DIF<sub>num</sub>: *more* and *less* denote the **difference function and its converse** (resp):

$$\text{more} \rightarrow \lambda m \lambda n. (n - m)$$

$$\text{less} \rightarrow \lambda m \lambda n. (m - n)$$

**STEP 2: COMPOSITION: REL<sub>num</sub>**

$$\emptyset_{a \text{ bit}} \text{ more} \rightarrow \lambda r. r > 0 \circ \lambda m \lambda n. (n - m)$$

$$\lambda r. r > 0 \circ \lambda m \lambda n. (n - m) = \lambda m \lambda n. ([\lambda r. r > 0](n - m)) = \lambda m \lambda n. (n - m) > 0$$

The scales have arithmetics, so we can use standard addition tricks:

$$\begin{aligned} \lambda m \lambda n. (n - m) > 0 &= \lambda m \lambda n. (n - m) + m > 0 + m = \\ \lambda m \lambda n. n > m &= >_{\mathbf{R}} \end{aligned}$$

$$\emptyset_{a \text{ bit}} \text{ more} \rightarrow >$$

$$\begin{aligned} \emptyset_{a \text{ bit}} \text{ less} &\rightarrow \lambda r. r > 0 \circ \lambda m \lambda n. (m - n) \\ &= \lambda m \lambda n. (m - n) > 0 = <_{\mathbf{R}} \end{aligned}$$

**STEP 3: PRED<sub>num</sub>: APPLY** the meaning of the REL<sub>num</sub> derived to 3:

$$\text{more than three} \rightarrow \lambda m. m >_{\mathbf{R}} 3$$

$$\text{less than three} \rightarrow \lambda m. m <_{\mathbf{R}} 3$$

**STEP 4: PRED<sub>unit</sub>: LIFT** the numerical predicate and cm to predicates of degrees, semantically adjoin the latter to the first:

We typeshift a predicate from a set of numbers to a set of degrees.

Let P be a predicate of numbers, u a unit

$$\text{LIFT}[P, u] = \lambda \delta. P(\delta^r) \wedge \delta^u = u$$

$$\text{more than three centimeters} \rightarrow \lambda \delta. \delta^r > 3 \wedge \delta^u = \text{cm}$$

the set of degrees whose numerical value is bigger than three and whose unit is cm

$$\text{less than three centimeters} \rightarrow \lambda \delta. \delta^r < 3 \wedge \delta^u = \text{cm}$$

the set of degrees whose numerical value is bigger than three and whose unit is cm

**Also:** null degree predicate  $\emptyset_{a \text{ bit}}$ : ( $\Delta$ : default unit)

$$\emptyset_{a \text{ bit}} \rightarrow \lambda \delta. \delta^r > 0 \wedge \delta^u = \Delta$$

**STEP 5 and 6: DIF<sub>unit</sub> and DIM:** *more*  $\rightarrow \lambda s. s^-$  *less*  $\rightarrow \lambda s. (s^c)^-$ 

$$\text{tall} \rightarrow \lambda u. \text{STALL}_{u,k} \quad \text{short} \rightarrow \lambda u. \text{STALL}_{u,k}^c$$

## STEP 7

$$\begin{array}{lcl}
\text{more than three centimeters} & + \text{ more} & = \text{ more than three centimeters more} \\
\text{one place predicate} & \text{3 place function} & \text{3 place relation} \\
\lambda\delta.\delta^r > 3 \wedge \delta^u = \text{cm} & \circ \quad \lambda s.s^- & = \\
& & \lambda s\lambda\delta_2\lambda\delta_1.s^-(\delta_1,\delta_2) \\
\lambda s\lambda\delta_2\lambda\delta_1. ([\lambda\delta.\delta^r > 3 \wedge \delta^u = \text{cm}] (s^-(\delta_1,\delta_2))) & & \\
= & & \\
\lambda s\lambda\delta_2\lambda\delta_1. s^-(\delta_1,\delta_2)^r > 3 \wedge s^-(\delta_1,\delta_2)^u = \text{cm} & & 
\end{array}$$

## STEP 8

$$\text{more than three centimeters more} + \text{tall} = \text{more than three centimeters more tall}$$

$$\text{tall} \rightarrow \lambda u.\text{STALL}_{,u}$$

At this point we use the fact that degrees have memory (are triples).

1. apply the meaning of *tall* to the unit extractable from the meaning of the three place relation between cm-degrees:  $\text{STALL}_{,\text{cm}}$
2. apply the three place relation to this:

$$\begin{array}{l}
\lambda\delta_2\lambda\delta_1. s^-(\delta_1,\delta_2)^r > 3 \wedge s^-(\delta_1,\delta_2)^u = \text{cm} (\text{STALL}_{,\text{cm}}) \\
= \\
\lambda\delta_2\lambda\delta_1. \text{STALL}_{,\text{cm}}^-(\delta_1,\delta_2)^r > 3
\end{array}$$

$$\text{Now: } \text{STALL}_{,\text{cm}}^- = \rightarrow_{\text{H}}$$

So:

$$\begin{array}{lcl}
\lambda\delta_2\lambda\delta_1. \text{STALL}_{,\text{cm}}^-(\delta_1,\delta_2)^r > 3 & = & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}. (\delta_1 \rightarrow_{\text{H}} \delta_2)^r > 3 \\
\lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}. \delta_1^r - \delta_2^r > 3 & = & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}. \delta_1^r > \delta_2^r + 3
\end{array}$$

$$\text{more than three centimeters more tall} = \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}. \delta_1^r > \delta_2^r + 3$$

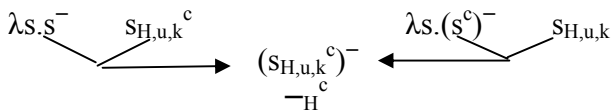
the relation that holds between two degrees if the numerical value of the first is more than the numerical value of the second plus 3.

After reduction:

$$\begin{array}{lcl}
[\text{REL}_{\text{dim}} \text{ more than three cm more tall than}] \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}: & \delta_1^r > \delta_2^r + 3 \\
[\text{REL}_{\text{dim}} \text{ less than three cm more tall than}] \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}: & \delta_1^r < \delta_2^r + 3 \\
[\text{REL}_{\text{dim}} \text{ more than three cm less tall than}] \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}: & \delta_1^r < \delta_2^r - 3 \\
[\text{REL}_{\text{dim}} \text{ less than three cm less tall than}] \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,cm}}: & \delta_1^r > \delta_2^r - 3 \\
[\text{REL}_{\text{dim}} \emptyset \text{ more tall than}] & \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,\Delta}}: \delta_1^r > \delta_2^r \\
[\text{REL}_{\text{dim}} \emptyset \text{ less tall than}] & \rightarrow & \lambda\delta_2\lambda\delta_1 \in \Delta_{\text{H,\Delta}}: \delta_1^r < \delta_2^r
\end{array}$$

**FACT:**  $\beta$  *more short* is equivalent to  $\beta$  *less tall*

**E.g.:** *at least three cm more short than* = *at least three cm less tall than*



Relations between individuals: composition with the measure function (twice in the derivation):

- |  |  |
|--|--|
| (1) <i>Fred is taller than Susan</i>                     | $H_{cm,w}(Fred)^r > H_{cm,w}(Susan)^r$     |
| (2) <i>Fred is more than three cm taller than Susan</i>  | $H_{cm,w}(Fred)^r > H_{cm,w}(Susan)^r + 3$ |
| (3) <i>Fred is less than three cm taller than Susan</i>  | $H_{cm,w}(Fred)^r < H_{cm,w}(Susan)^r + 3$ |
| (4) <i>Fred is shorter than Susan</i>                    | $H_{cm,w}(Fred)^r < H_{cm,w}(Susan)^r$     |
| (5) <i>Fred is more than three cm shorter than Susan</i> | $H_{cm,w}(Fred)^r < H_{cm,w}(Susan)^r - 3$ |
| (6) <i>Fred is less than three cm shorter than Susan</i> | $H_{cm,w}(Fred)^r > H_{cm,w}(Susan)^r - 3$ |

Case (3) (and 6): compare (7):

- (7) A. Is John taller than Mary?  
 B. I don't know. But I *do* know that he is less than two centimeters taller than Mary. You see, Mary is 1.63. And I happen to know that John was rejected by the police because of his height, and they only accept people 1.65 and up.

This discourse is felicitous and compatible with John being **smaller** than Mary, supporting the interpretation given in (3).

### This semantics for DP comparatives is a generalization of Hoeksema 1982

#### Montague's generalization applies to DP comparatives:

The DP complement of an extensional transitive verb/DP-comparative relation **takes semantic scope over** the meaning of the transitive verb/comparative relation.

DP<sub>1</sub> TV DP<sub>2</sub>: Normal scope reading: DP<sub>1</sub>( $\lambda x. DP_2(\lambda y. TV(x,y))$ )

1. Shift *taller than* by composition with the measure function:

$$\lambda \delta_2 \lambda \delta_1 \in \Delta_{H,\Delta}: \delta_1^r > \delta_2^r \text{ shifts to } \lambda y \lambda \delta_1 \in \Delta_{H,\Delta}: \delta_1^r > H_\Delta(x,w)^r$$

2. We apply to the object DP:

$$\lambda \delta_1 \in \Delta_{H,\Delta} DP_2 (\lambda y: \delta_1^r > H_\Delta(y,w)^r)$$

3. Shift this with the measure function:

$$\lambda x. DP_2 (\lambda y: H_\Delta(x,w)^r > H_\Delta(y,w)^r)$$

4. Apply this to the subject DP:

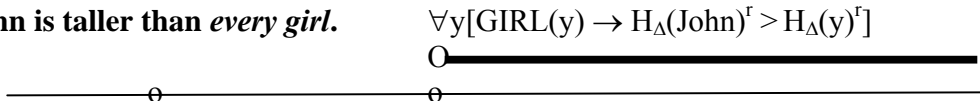
$$DP_1(\lambda x. DP_2 (\lambda y: H_\Delta(x,w)^r > H_\Delta(y,w)^r))$$


**Prediction:** DP comparatives are semantically like extensional transitive verbs.


#### Support 1: scope of quantificational DP complements

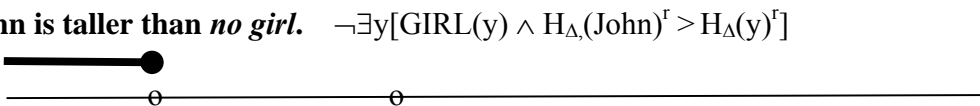



**QUANTIFICATIONAL DP COMPLEMENTS** (ignoring world parameter w)


**John is taller than every girl.**  $\forall y[\text{GIRL}(y) \rightarrow H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$   
  
 $H_{\Delta}(g_1)^r \dots\dots\dots H_{\Delta}(g_n)^r$  **John is taller than the tallest girl.**

**John is taller than some girl.**  $\exists y[\text{GIRL}(y) \wedge H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$   
  
 $H_{\Delta}(g_1)^r \dots\dots\dots H_{\Delta}(g_n)^r$  **John is taller than the shortest girl.**

**John is taller than exactly three girls.**  $|\text{GIRL} \cap \lambda y. H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r| = 3$   
  
 $H_{\Delta}(g_1)^r H_{\Delta}(g_2)^r H_{\Delta}(g_3)^r H_{\Delta}(g_4)^r \dots\dots\dots H_{\Delta}(g_n)^r$   
**John is taller than the shortest three girls, but not taller than any other girls.**  
 (Many theories have problems getting this)

**John is taller than no girl.**  $\neg \exists y[\text{GIRL}(y) \wedge H_{\Delta}(\text{John})^r > H_{\Delta}(y)^r]$   
  
 $H_{\Delta}(g_1)^r \dots\dots\dots H_{\Delta}(g_n)^r$  **John's height is at most that of the shortest girl.**  
 (Stilted, because English prefers auxiliary negation, but felicitous.)

**John is at least two cm taller than every girl.**  $\forall y[\text{GIRL}(y) \rightarrow H_{\text{cm}}(\text{John})^r \geq H_{\text{cm}}(y)^r + 2]$   
  
 $H_{\Delta}(g_1)^r + 2 \dots\dots\dots H_{\Delta}(g_n)^r + 2$   
 John's height is at least the height of the **tallest** girl plus two inches.

**John is at most two cm taller than every girl.**  $\forall y[\text{GIRL}(y) \rightarrow H_{\text{cm}}(\text{John})^r \leq H_{\text{cm}}(y)^r + 2]$   
  
 $H_{\Delta}(g_1)^r + 2 \dots\dots\dots H_{\Delta}(g_n)^r + 2$   
 John is at most the height of the **shortest** girl plus two cm.

- Cf. the following valid inference:
- John is at most two cm taller than every girl.
  - Mary is the shortest girl.
  - Hence, John is at most two cm taller than Mary.

# LECTURE 3

## DP-comparatives:

(1) Donald is richer than [DP Fred]

## CP-comparatives:

(2) Donald is richer than [CP Fred has ever been –]

More general characterization:

## Categorical comparatives: [XP $\alpha$ ] is comp than [XP $\beta$ ]

-comparative relations between expressions of the same categories:

(1) [DP Donald] is richer than [DP Fred]

(3) [INF to be] is more interesting than [INF not to be]

(4) [S Jim runs] faster than [S you run]

With ellipsis:

(5) Jim runs faster than you

either: a. **extended comparative relation:**

[DP Jim] [<sub>compar</sub> runs faster ] than [DP you]

or: b. **categorical comparative with ellipsis:**

[S Jim runs] faster than [S you – ]  
run

## CP-comparatives: [XP $\alpha$ ] is comp than [CP ...[XP $\beta$ ] –...]

$\lambda\delta$                        $\delta$

Semantically interpreted gap-construction in which an comparative XP occurs, but which can freely contain other CP internal material:

(6) Donald is richer than [CP Fred has ever been –]  
[CP Every Englishman might have been – in 1989 ]

We cannot exclude the possibility that DP comparatives also have readings as elliptical CP-comparative.

(1) Donald is richer than [DP Fred]

(7) Donald is richer than [CP [DP Fred] –]

-Semantics from DP-comparatives given can be extended without much problem to categorical comparatives.

-But what is the semantics for CP comparatives?

-If DP comparatives are a simple case of CP-comparatives with ellipsis, then we don't need the DP comparatives given.

-While we cannot exclude the possibility that DP comparatives have readings as elliptical CP-comparatives, we can show that not all DP comparatives (i.e. not all categorical comparatives) are elliptical CP-comparatives.

### Argument 1: Classical argument of anaphora.

- (1) a. Nobody is taller than himself.  
[<sub>DP</sub> Nobody] is taller than [<sub>DP</sub> *himself*]
- b. #Nobody is taller than himself is.  
[<sub>DP</sub> Nobody] is taller than [<sub>CP</sub> [<sub>DP</sub> *himself*] is –]
- c. #Nobody is taller than himself.  
[<sub>DP</sub> Nobody] is taller than [<sub>CP</sub> [<sub>DP</sub> *himself*] – –]

If (1a) has only an analysis as an elliptical CP comparative, it is incorrectly predicted to be infelicitous.

### Argument 2: Downward entailing DPs in CP-comparatives.

#### Hoeksema, Rullmann: Downward entailing DPs in CP-complements are infelicitous.

- (2) a. Mary is taller than nobody.
- b. #Mary is taller than nobody is –.
- c. #Mary is taller than nobody ever was –.

Cf, also

- (3) a. #Mary is more famous than John isn't –.
- b. #Mary is more famous than John will never be –.
- c. #Bill is taller than at most three girls ever were –.
- d. #Bill is at least two inches taller than nobody ever was.
- e. #Bill is at most two inches taller than nobody ever was.
- f. #Bill is exactly two inches taller than nobody every was.

-(2a) is felicitous, if stilted, and *nobody* has a wide scope reading:

Nobody is such that Mary is taller than them., i.e. Mary is the shortest.

-(2b) and (2c) are baffling.

What does (2c) mean? Are you trying to say that nobody ever was as tall as Mary is? Mary's height has boldly gone where nobody's height has ever gone before?

My own judgement: My brain is trying several interpretation strategies simultaneously and gets hopelessly muddled.

If (2a) has only an interpretation as an elliptical CP comparative, it is predicted to be infelicitous.

**Argument 3: A cross-linguistic argument:** Mandarin Chinese has categorial comparatives, with ellipsis, but no CP comparatives.

Comparative data in Li and Thompson 1981:

Comparatives of the form: (° is the position where in *not as...that... (less ... than)* or *with...same... (as...as...)* you find the second element

I **more** You ° gradable predicate: *tall/like Susan/come early/late eat food*  
I am taller/like Susan more/come earlier/...than you

I run **more** you run ° *fast*

I run **more** you – ° *fast*  
run

I run faster than you

Elephant – **more** bear nose ° *long*  
nose

An elephant has a longer nose than a bear

I – **more** – yesterday ° *comfortable*  
now I

I am more comfortable (now) than (I was) yesterday

But (Lise Cheng, p.c.) Mandarin Chinese does not have CP comparatives.

Thus we do not find:

#I **not as** you might have been **that** rich

I am less rich than you might have been

#I **not as** every member of my club was in 1998 **that** rich

I am less rich than every member of my club was in 1998

If DP comparatives are elliptical CP comparatives, then there are only CP comparatives, and one would expect CP comparatives to be universal.

On the other hand, if Chinese has categorial comparatives, why not English as well.

#### Argument 4: DP-comparatives and polarity.

DP-comparatives **seem** to allow polarity sensitive (PS) items and seem to be downward entailing (DE):

- (1) a. Mary is more famous than *anyone*.  
b. (1) Mary is more famous than John *or* Bill.  
(2) Hence, Mary is more famous than John.

#### Hoeksema 1982:

1. *anyone* in (1a) and *or* in (1b) allow **free choice interpretations** (FC).

**Hence:** the facts in (1) are consequences of FC interpretations, not PS interpretations. (Certainly DP-comparatives allow FC-*any*: only FC-*any* can be modified by *almost*

- (2) Mary is more famous than *almost anyone*.)

2. **DP comparatives are not downward entailing:** (3) is **invalid**:

- (3) (1) John is more famous than *Mary*.  
(2) *Mary* is a girl.  
(3) Hence, John is more famous than *every girl*.

3. **Semantics of DP comparatives predicts that polarity items are not licensed**, since by Montague's generalization, the DP complement takes semantic scope over the comparative relation.

4. **Dutch has PS items that are not FC items, and these are not felicitous in DP comparatives.**

Hoeksema: *ook maar iemand* is PS but not FC (cf. FC item *wie ook maar*):

- (4) a. **Ik leen geen boeken uit aan *ook maar iemand*.**      **DE context:**  
I lend no books out to *ook-maar*-someone      **PS felicitous**  
I don't lend books to anyone
- b. **#Dat kan je *ook maar iemand* vragen.**      **Modal context:**  
That can you *ook-maar*-someone ask      **PS infelicitous**  
That, you can ask anyone
- c. **Dat kan je *wie dan ook* vragen.**      **Modal context:**  
That can you who-*dan ook* ask      **FC felicitous**  
That, you can ask anyone

PS items are felicitous in CP comparatives but not in DP comparatives:

- (5) a. **Marie is beroemder dan *ook maar iemand* ooit geweest is.**      **CP comparative**  
Marie is more famous than *ook-maar*-someone ever been is      **PS felicitous**  
Marie is more famous than anyone has ever been.
- b. **#Marie is beroemder dan *ook maar iemand*.**      **DP comparative**  
Marie is more famous than *ook-maar*-someone      **PS infelicitous**  
Marie is more famous than anyone
- c. **Marie is beroemder dan *wie dan ook*.**      **DP comparative**  
Marie is more famous than who-*dan ook*      **FC felicitous**  
Marie is more famous than anyone

Hoeksema's claim can be strengthened by looking at stressed *énige*. As a plural, not-necessarily stressed item *enige* means *a few*, and is not at all a polarity item:

- (6) Ik heb hem *enige boeken* uitgeleend.  
I lent him *a few* books.

But as a singular, stressed element, *énige* is a PS item, and it means *any*, PS *any*, and not FC *any*:

- (7) a. **Ik leen geen boeken uit aan *énige filosoof*.** DE context  
I lend no books to any philosopher PS felicitous  
b. **#Dat kan je *énige filosoof* vragen.** UE modal context:  
That, you can ask any philosopher. PS infelicitous

And we find that *énige* is infelicitous in DP comparatives:

- (8) a. **Marie is beroemder dan *énige filosoof* ooit geweest is.** CP comparative  
Marie is more famous than an philosopher has ever been. PS felicitous  
b. **#Marie is beroemder dan *énige filosoof*** DP comparative  
Marie is more famous than any philosopher PS infelicitous

### Complicating data:

(5b) and (8b) **improve** in felicity if we tag on them a **FC appositive phrase**:

- (9) a. Marie is beroemder dan *ook maar iemand, wie dan ook*.  
b. Marie is beroemder dan *énige filosoof, welke je ook maar kiest*.  
whichever one you choose.

This supports: FC is licensed in DP-comparatives, PS is not.

The data are real, but not as strong for every speaker.

Crit Cremers p.c.: (10b) is fine or almost fine:

- (10) a. #Marie is beroemder dan *énige filosoof*  
b. Marie is beroemder dan *welhaast enige filosoof*  
Mary is more famous than almost any philosopher.

*Madame Bovary*, translation Hans van Pinxteren:

- (11) Een boerenjongen van een jaar of vijftien, een stuk groter dan *één van ons*  
A farmers boy of about fifteen, a lot bigger than any of us

van Pinxteren finds this good enough to show on the first page of *Madame Bovary*, while for me this is completely infelicitous.

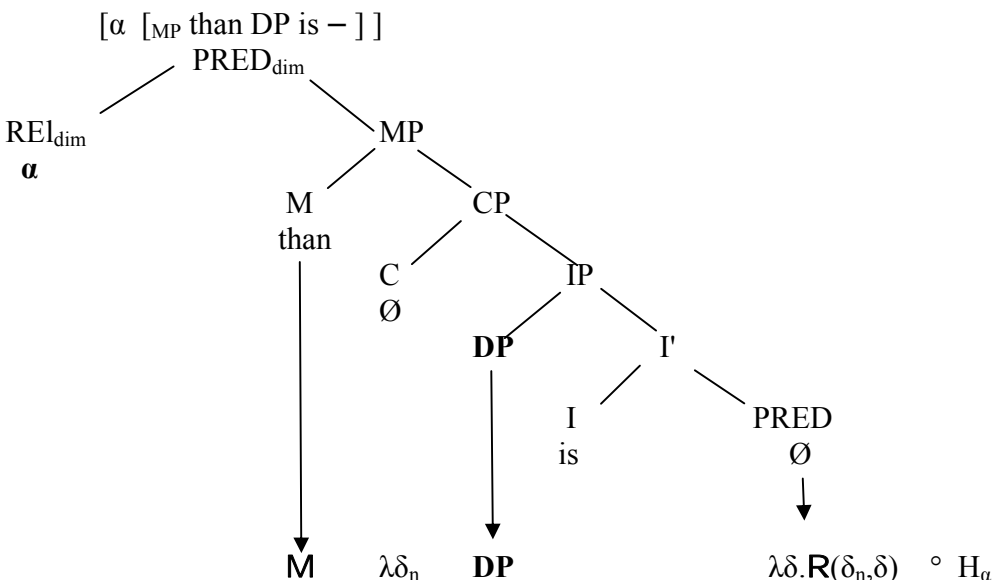
Where is the variation to be located? Suggestion:

- Elliptic CP-complement readings do exist, and license polarity items
- Competition between DP-complement reading and elliptical CP-complement reading is slanted in favour of DP-complement reading.
- What varies is how accessible the CP-complement reading is:
  - For me: difficult
  - For Crit: easier
  - For van Pinxteren: easy

# GENERAL SEMANTICS FOR CP COMPARATIVES

Terminology: DP-comparative and CP-comparative in (1): **comparative correlates**.

- (1) a. John is taller than DP  
 b. John is taller [<sub>CP</sub> than DP is – ]



$\text{REL}_{\text{dim}} \alpha$  is the relation we have built up for the DP comparative.

**1. Classical wisdom:** The CP contains syntactically an operator-gap construction.

**2. Standard assumption:** this construction is semantically interpreted as variable binding.

-We introduce a variable  $\delta_n$  at the gap

-We abstract over this variable at the CP-level:  $\lambda\delta_n \dots \delta_n \dots$

**3. Common assumption:** The variable abstracted over is a **degree variable**.

**4. Common assumption:** the gap is a predicate gap., it is interpreted as a predicate

**5. Modularity:** the gap denotes a degree predicate.  $\lambda\delta \dots \delta \dots$

Since the gap **contains** degree variable  $\delta_n$  that is abstracted over at the CP level, the gap is interpreted as a degree predicate:

**Gap:**  $\lambda\delta.\mathbf{R}(\delta_n, \delta)$  for some relation  $\mathbf{R}$

inverse notation:  $\lambda\delta. \delta_n \mathbf{R} \delta$

**6. Modularity:**  $\text{DP} + \lambda\delta. \delta_n \mathbf{R} \delta$ : shift the degree predicate to a predicate of individuals by composition with the measure function.

7. **Rich degrees:** the degree predicate  $\lambda\delta. \delta_n \mathbf{R} \delta$  will be constrained by the external degree relation  $\alpha$ . From  $\alpha$  (and the context) we can reconstruct the measure function  $M_{\alpha,k}$ . Hence the gap shifts to predicate of individuals:

**Gap:**  $\lambda y. \delta_n \mathbf{R} M_{\alpha,k}(y, w)$

8. **IP semantics:**

$DP(\lambda y. \delta_n \mathbf{R} M_{\alpha,k}(y, w))$

9. **Abstraction at the level of the CP:**

$\lambda\delta_n. DP(\lambda y. \delta_n \mathbf{R} M_{\alpha,k}(y, w))$

$\lambda\delta. DP(\lambda y. \delta \mathbf{R} M_{\alpha,k}(y, w))$

This gives a set of degrees as the denotation of the complement so far. Some theories in the literature assume that at this level an implicit operation takes place (like maximalization). We assume an operation  $\mathbf{M}$  in order to compare theories with each other:

10. **CP semantics:**  $\mathbf{M}(\lambda\delta. DP(\lambda y. \delta \mathbf{R} M_{\alpha,k}(y, w)))$

11. Dimensionalrelation  $\alpha + \text{CP}$ :

**GENERAL SEMANTICS FOR COMPARATIVE COMPLEMENTS:**

$[\alpha \quad [_{\text{MP}} \text{ than DP is } - ] \quad ]$

$\alpha + \mathbf{M}(\lambda\delta. DP(\lambda y. \delta \mathbf{R} M_{\alpha,k}(y, w)))$

1. **What is relation  $\mathbf{R}$ ?**

2. **What is operation  $\mathbf{M}$ ?**



## Theory I: Von Stechow 1984:

**R** = (identity)

**M**  $\sqsubset_{<}$  (supremum, maximalization operation)

Thus:

*taller + than DP is –*

$>_H$  **M** ( $\lambda\delta.DP(\lambda y.\delta R H_{\Delta,k}(y,w))$ )

$\sqsubset_{<H}(\lambda\delta.DP(\lambda y.\delta = H_{\Delta,k}(y,w)))$

*taller than DP is:*

$\lambda\delta. \delta >_H \sqsubset_{<H}(\lambda\delta.DP(\lambda y.\delta = H_{\Delta,k}(y,w)))$

*taller than Mary is*

$\lambda\delta. \delta >_H \sqsubset_{<H}(\lambda\delta. \lambda P.P(m) (\lambda y.\delta = H_{\Delta,k}(y,w)))$

=

$\lambda\delta. \delta >_H \sqsubset_{<H}(\{H_{\Delta,k}(m,w)\}) = \lambda\delta. \delta >_H H_{\Delta,k}(m,w)$

The set of height-degrees bigger than Mary's height

*taller than every girl is*

$\lambda\delta. \delta >_H \sqsubset_{<H}(\lambda\delta. \lambda P.\forall y[GIRL(y) \rightarrow P(y)](\lambda y.\delta = H_{\Delta,k}(y,w)))$

=

$\lambda\delta. \delta >_H \sqsubset_{<H}(\lambda\delta. \forall y[GIRL(y) \rightarrow \delta = H_{\Delta,k}(y,w)])$

The set of degrees that are identical to the height of every girl is only defined if all girls are of the same height, and then it denotes the set of that height.

Thus:

*Bill is taller than every girl is*

presupposes, for von Stechow, that all girls have the same height.

*taller than some girl is*

$\lambda\delta. \delta >_H \sqsubset_{<H}(\lambda\delta. \exists y[GIRL(y) \wedge \delta = H_{\Delta,k}(y,w)])$

=

$\lambda\delta. \delta >_H \sqsubset_{<H}(\{H_{\Delta,k}(y,w): y \in GIRL\})$

$\{H_{\Delta,k}(y,w): y \in GIRL\}$  is the set of all girl-height degrees.

$\sqsubset_{<H}(\{H_{\Delta,k}(y,w): y \in GIRL\})$  is the height of the tallest girl.

Thus:

*Bill is taller than some girl is*

means, for von Stechow:

Bill is taller than the tallest girl.

## Theory II: Reconstruction of Heim 1980:

|          |  |                                      |
|----------|--|--------------------------------------|
| <b>R</b> | $\lambda\delta_2\lambda\delta_1. 0 < \delta_1^r \leq \delta_2^r$ | (monotonic closure down)             |
| <b>M</b> | $\sqcup_{<}$   | (supremum, maximalization operation) |

*taller + than DP is –*

$$\begin{aligned} >_H \quad \mathbf{M} \quad (\lambda\delta. DP(\lambda y. \delta \mathbf{R} H_{\Delta,k}(y,w))) \\ \sqcup_{<H} (\lambda\delta. DP(\lambda y. 0 < \delta_r < H_{\Delta,k}(y,w)^r)) \end{aligned}$$

*taller than DP is:*

$$\lambda\delta. \delta >_H \sqcup_{<H} (DP(\lambda y. 0 < \delta_r < H_{\Delta,k}(y,w)^r))$$

*taller than Mary is*

$$\lambda\delta. \delta >_H \sqcup_{<H} (\lambda\delta. \lambda P. P(m) \lambda y. 0 < \delta_r < H_{\Delta,k}(y,w)^r)$$

$$=$$

$$\lambda\delta. \delta >_H \sqcup_{<H} (\lambda\delta. 0 < \delta_r < H_{\Delta,k}(m,w)^r) = \lambda\delta. \delta >_H H_{\Delta,k}(m,w)$$

The maximum of the set of heights between 0 and Mary's height is Mary's height.

Hence:

The set of height-degrees bigger than Mary's height

*taller than every girl is*

$$\lambda\delta. \delta >_H \sqcup_{<H} (\lambda\delta. \forall y [GIRL(y) \rightarrow 0 < \delta_r < H_{\Delta,k}(y,w)^r])$$

$\lambda\delta. \forall y [GIRL(y) \rightarrow 0 < \delta_r < H_{\Delta,k}(y,w)^r]$  is the set of degrees that bigger than 0 but lower than the height of every girl.

This is the set of all degrees bigger than 0 below the height of the smallest girl.

Hence, the supremum of this set is the height of the smallest girl.

Hence:

*Bill is taller than every girl is*

means, for Heim: Bill is taller than the smallest girl.

*taller than some girl is*

$$\lambda\delta. \delta >_H \sqcup_{<H} (\lambda\delta. \exists y [GIRL(y) \wedge 0 < \delta_r < H_{\Delta,k}(y,w)^r])$$

$\lambda\delta. \exists y [GIRL(y) \wedge 0 < \delta_r < H_{\Delta,k}(y,w)^r]$  is the set of height degrees bigger than 0 but smaller than the height of the tallest girl.

Hence,  $\sqcup_{<H} (\lambda\delta. \exists y [GIRL(y) \wedge 0 < \delta_r < H_{\Delta,k}(y,w)^r])$  is the height of the tallest girl.

Thus:

*Bill is taller than some girl is*

means, for Heim: Bill is taller than the tallest girl.

## VON STECHOW'S SUPREMUM THEORY.

### PROBLEM: (Schwarzschild and Wilkinson):

Maximalization theories predict unnatural readings and fail to predict natural readings.

(1) John is taller than some girl is –

**von Stechow: Wrong meaning:** John is taller than the **tallest girl**

(2) John is taller than every girl is –

**Heim: Wrong meaning:** John is taller than the **shortest girl**

**von Stechow:** John is taller than the degree to which every girl is tall.

**Unwarranted presupposition:** all girls have the same degree of height

### PROPOSED SOLUTION (Larsons, Von Stechow):

**Give the CP internal DP scope over the comparative.**

#### Problem 1:

This requires **systematic scoping of all kinds of DPs out of the CP**, which is problematic.

Cf. scoping out of relative clauses, wh-clauses, or even propositional attitude complements: in all other CPs scoping out is severely restricted. (Larsons and von Stechow propose non-standard scope mechanisms: a recognition of the problem, but not a solution.)

#### Problem 2: Intensional contexts (Schwarzschild and Wilkinson)

(3) John is taller than [<sub>CP</sub> Bill believes that every girl in Dafna's class is –.]

$\lambda\delta. \text{BELIEVE}(\text{BILL}, \wedge \forall x[\text{GIRL-IN-DC}(x) \rightarrow \delta^f > H_{\Delta}(x)^f]) (H_{\Delta}(\text{John}))$

John's actual height has the property that Bill believes it to be bigger than what he thinks is the height of what he thinks is the tallest girl in Dafna's class.

-No presupposition that Bill believes that the girls have the same height.

-von Stechow: scope *every girl* over the comparative.

-But *every girl* takes narrow scope under *believe*, which is inside the comparative.

#### Problem 3: Polarity items:

Unlike DP-comparatives, CP-comparatives allow polarity items inside the CP-complement:

(4) **Marie is beroemder dan enige filosoof en enige psycholoog ooit geweest zijn.**

Marie is more famous than any philosopher and any psychologist have ever been.

-No presupposition that any philosopher has the same degree of fame as any psychologist.

- von Stechow: scope *enige filosoof en enige psycholoog* over the comparative.

-But conjunction of PS items: **not licensed if scoped over the comparative.**

### CONCLUSION (Schwarzschild and Wilkinson):

**Supremum theories like von Stechow's (and Heim's) are untenable.**

# LECTURE 4

## THE INTERVAL THEORY OF SCHWARZSCHILD AND WILKINSON

**DP<sub>1</sub> is  $\alpha$  than DP<sub>2</sub> is –**

Schwarzschild and Wilkinson:

**DP<sub>1</sub> is  $\beta$ -taller than DP<sub>2</sub> is –.**

where  $\beta$  is a numerical predicate of the form: *at least two centimeters, at most two centimeters, exactly two centimeters,  $\emptyset$ .....*

$\exists j [ \text{DP}_1 \text{ is } j\text{-tall} \wedge \text{DP}_2 \text{ is } \max(\lambda i. \beta(j-i))\text{-tall} ]$

There is a degree interval  $j$  such that DP<sub>1</sub> is  $j$ -tall and DP<sub>2</sub> is  $k$ -tall, where:

$k$  is the maximal interval in the set of intervals  $i$  such that  $j-i$  is  $\beta$

We will use an example:

(1) John is more than 2 centimeters taller than exactly four girls are.

$\exists j [ \text{John is } j\text{-tall} \wedge \text{Exactly four girls are } \max(\lambda i. \text{more than 2 cm}(j-i))\text{-tall} ]$

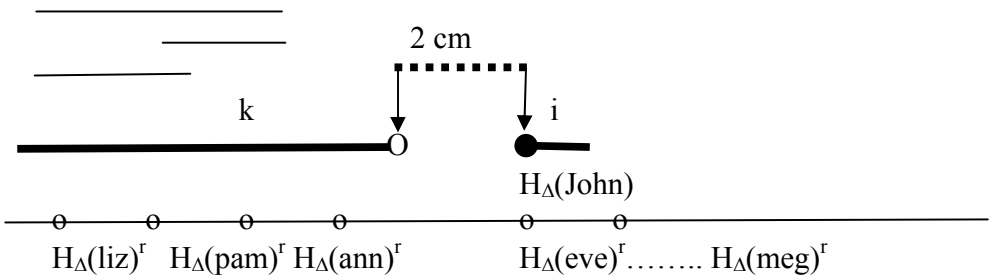
### Intuition:

"If a comparative statement is true, we should be able to perform the following routine. First we show some interval  $[j]$  that satisfies the main clause [*John is  $j$ -tall*].

Then we find the largest interval  $[k]$  all of whose parts are below  $[j]$  by the amount given by the differential [*more than 2 cm*].

We then show that that maximal interval satisfies the subordinate clause [*exactly four girls are  $k$ -tall*]" (Schwarzschild and Wilkinson, p. 23)

intervals more than 2 cm below



## From interval degrees to point degrees.

1. We are not concerned with  $DP_1$ :

**$DP_1$  is  $\beta$ -taller than  $DP_2$  is  $-$ .**

$DP_1(\lambda x. \exists j [x \text{ is } j\text{-tall} \text{ and } DP_2 \text{ is } \max(\lambda i. \beta([j-i])\text{-tall})])$

**be  $\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda x. \exists j [x \text{ is } j\text{-tall} \text{ and } DP_2 \text{ is } \max(\lambda i. \beta([j-i])\text{-tall})]$

2.  $x$  is  $j$ -tall means:  $H_{cm}(x) \in j$ . This is too weak, we will correct it to:

**be  $\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda x. \exists j [H_{cm}(x) = j \text{ and } DP_2 \text{ is } \max(\lambda i. \beta([j-i])\text{-tall})]$

(technically:  $j = [H_{cm}(x), H_{cm}(x)]$ , but we set  $[r,r] = r$ ).

**be  $\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda x. DP_2 \text{ is } \max(\lambda i. \beta([H_{cm}(x)-i])\text{-tall})$

3. We get a degree predicate by inverting composition with the measure function:

$\lambda x. [\lambda \delta. DP_2 \text{ is } \max(\lambda i. \beta([\delta-i])\text{-tall})] (H_{cm}(x))$

**$\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda \delta. DP_2 \text{ is } \max(\lambda i. \beta([\delta-i])\text{-tall})$

4. Interpretative assumption:

$DP_2 \text{ is } P$  is interpreted as  $DP_2(\lambda y. P(y))$

Hence:  **$\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda \delta. DP_2(\lambda y. y \text{ is } \max(\lambda i. \beta([\delta-i])\text{-tall}))$

5. As before,  $y$  is  $j$ -tall means  $H_{cm}(y) \in j$ , so:

**$\beta$ -taller than  $DP_2$  is  $-$ .**

$\lambda \delta. DP_2(\lambda y. H_{cm,w}(y) \in \max(\lambda i. \beta(\delta-i)))$

6.  $\lambda \delta. DP_2(\lambda y. H_{cm,w}(y) \in \max(\lambda i. \beta(\delta_1-i)))$

$\lambda \delta. DP_2(\lambda y. \underline{[\lambda \delta_2 \lambda \delta_1. \delta_2 \in \max(\lambda i. \beta(\delta_1-i))]}(\delta, H_{cm}(y)))$

**$R(\delta, H_{cm}(y))$**

where  **$R = [\lambda \delta_2 \lambda \delta_1. \delta_2 \in \max(\lambda i. \beta(\delta_1-i))]$**

7.  **$\beta$ -taller than DP<sub>2</sub> is  $-$ .**  
 $\lambda\delta.DP_2(\lambda y. \mathbf{R}(\delta, H_{cm}(y)))$  where  $\mathbf{R} = [\lambda\delta_2\lambda\delta_1. \delta_2 \in \max(\lambda i. \beta(\delta_1-i))]$

With this, we have fitted Schwarzschild and Wilkinson in the schema for the semantics for CP comparatives.

But we can go further.

8. Landman 2010 (AC paper) proves:

**Lemma:**  $\delta_2 \in \max(\lambda i. \beta(\delta_1-i))$  iff  $\beta(\delta_1^r -_R \delta_2^r)$

**Proof:** see Landman 2010.

8. With the lemma:

**$\beta$ -taller than DP<sub>2</sub> is  $-$ .**

$\lambda\delta.DP_2(\lambda y. \mathbf{R}(\delta, H_{cm}(y)))$  where  $\mathbf{R} = \lambda\delta_2\lambda\delta_1. \beta(\delta_1^r -_R \delta_2^r)$

- 9 One more equivalence: (using the assumption that when Schwarzschild and Wilkinson use *more than*, *less than*, ... they mean the same by that as what I mean by that):

$\beta(\delta_1^r -_R \delta_2^r)$  iff  $\beta \circ -_H(\delta_1, \delta_2)$

**$\beta$ -taller than DP<sub>2</sub> is  $-$ .**

$\lambda\delta.DP_2(\lambda y. [\beta \circ -_H](\delta_1, H_{cm}(y)))$

10. In the modular theory:  **$\beta$ -taller** is interpreted as:  $\beta \circ -_H$

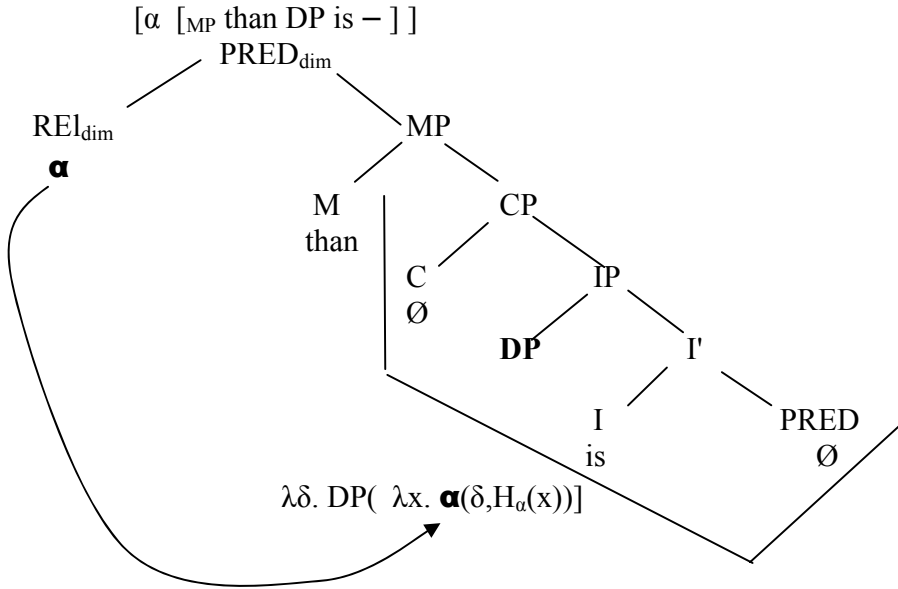
With this we conclude:

### THEORY III:

**SWL: Schwarzschild and Wilkinson, as reduced in Landman 2010 (AC 2009)**

**R**                     $\alpha$                     (the external comparative relation)

**M**                     $\lambda P.P$                     (identity)



**α is interpreted inside the comparative CP as part of the interpretation of the gap.**

(5) John is taller than Mary/ every girl is –

**Everybody else:** The CP denotes a set of degrees to which Mary, every girl is tall.  
**Schwarzschild and Wilkinson:** The CP denotes the set of degrees **bigger than** Mary’s height/every girl’s height.

**GOOD PREDICTIONS:**

**1. Correct readings:**

CP-comparatives that have correlates have the same interpretation as their correlates:

John is taller than Mary is:

$$\alpha = >_H$$

$$\lambda PP(j) (\lambda x. [\lambda \delta. \lambda PP(m) (\lambda y. \delta >_H H_{\Delta}(y))] (H_{\Delta}(x)) )$$

=

$$\lambda PP(m) (\lambda y. H_{\Delta}(j) >_H H_{\Delta}(y)) = H_{\Delta}(j) >_H H_{\Delta}(m)$$

The point is: by interpreting the external degree relation internally, you avoid the problem of scoping the internal DP out, and the internal DP relation takes scope over the degree relation: hence:

John is taller than every girl is:

=

$$\lambda P \forall y [GIRL(y) \rightarrow P(y)] (\lambda y. H_{\Delta}(j) >_H H_{\Delta}(y))$$

$$\forall y [GIRL(y) \rightarrow H_{\Delta}(j) >_H H_{\Delta}(y)]$$

John is taller than the tallest girl

John is taller than some girl is:

=

$\lambda P \exists y [\text{GIRL}(y) \wedge P(y)] (\lambda y. H_{\Delta}(j) >_H H_{\Delta}(y))$

$\exists y [\text{GIRL}(y) \wedge H_{\Delta}(j) >_H H_{\Delta}(y)]$

John is taller than the shortest girl

*exactly three girls*  $\rightarrow \lambda P. |\text{GIRL} \cap P| = 3$

John is taller than exactly three girls are:

$|\text{GIRL} \cap \lambda y. H_{\Delta}(\text{John}) >_H H_{\Delta}(y)| = 3$

John is taller than the third smallest girl, but not taller than any other girls

## 2. No unwanted presuppositions:

*John is taller than every girl* does not presuppose that all girls have the same height.

*Mary is more famous than any philosopher and any linguist has ever been* does not presuppose that any philosopher ever had the same (maximal) degree of fame as any linguist:

Consequently, **no scoping** out of the CP is necessary to get the correct readings.

## TWO PROBLEMS FOR SWL

### PROBLEM ONE: Polarity items in CP-comparatives.

PS items are felicitous in CP-comparatives, but CP comparatives are not downward entailing (Schwarzschild and Wilkinson):

- (3) a. John is more famous than *Mary* is –.
- b. Mary is a girl.
- c. Hence, John is more famous than *every girl* is.

**SWL has no stage in the semantic derivation on which to hang the difference in polarity licensing between CP comparatives and DP comparatives.**

### PROBLEM TWO: downward entailing DPs in CP-comparatives.

**Hoeksema, Rullmann: Downward entailing DPs in CP-complements are infelicitous.**

- (1) a. Mary is taller than nobody.
- b. **#**Mary is taller than nobody is –.
- c. **#**Mary is taller than nobody ever was –.

**SWL makes a wrong prediction:**

- (1b) means the same as (1a).



# THE DIRECTION OF COMPLEX RELATIONS

Back to the semantics of DP comparatives.

I have introduced comparison scales with different domains but intuitively the same direction:  $>_R$  and  $>_M$ , and comparison scales with the same domain but different directions:

$>_M$  and  $<_M$ .

+ : a direction for relations in the same direction as the basic scale

- : a direction of a relation in the same direction as the converse scale.

and we generalize this to associated functions:

+ :  $>_R, \geq_R, -_R, >_H, \geq_H, -_H$     *more<sub>num</sub>, at least<sub>num</sub>, more<sub>unit</sub>, tall*

- :  $<_R, \leq_R, -_R^c, <_H, \leq_H, -_H^c$     *less<sub>num</sub>, at most<sub>num</sub>, less<sub>unit</sub>, short*

(We will, for simplicity ignore the cases involving  $=_M$ , the discussion is easily amended to them: they can count as both + and -.)

All complex relations  $\alpha$  derived have a normal form:

$\lambda\delta_2\lambda\delta_1. \mathbf{R}_\alpha(\delta_1, \mathbf{f}(\delta_2))$     where  $\mathbf{R}_\alpha \in \{>_M, \geq_M, <_M, \leq_M, =_M\}$   
and  $\mathbf{f}$  is a differential (like  $+ <3, u, M>$ )

( e.g.  $\alpha = \lambda\delta_2\lambda\delta_1. \delta_1 \leq_M(\delta_2 -_M <3, u, M>)$  )  
 $\mathbf{R}_\alpha$                        $\mathbf{f}$

We will set:

**The direction (+/-) of complex relation  $\alpha$  is the direction of  $\mathbf{R}_\alpha$**

**FACT: The direction of the complex relation  $\alpha$  is predictable from the derivation by:**

**the direction of  $\alpha$  is + if the number of - elements used in the derivation is even,  
the direction of  $\alpha$  is - if the number of - elements used in the derivation is odd.**

|                               |                        |  |
|-------------------------------|------------------------|--|
| <i>more than three inches</i> | <i>more tall than</i>  | $\delta_1^r \triangleright_R \delta_2^r + 3$ |
| +                             | + +                    | +  |
| <i>less than three inches</i> | <i>more tall than</i>  | $\delta_1^r \triangleleft_R \delta_2^r + 3$  |
| -                             | + +                    | -  |
| <i>less than three inches</i> | <i>more short than</i> | $\delta_1^r \triangleright_R \delta_2^r - 3$ |
| -                             | + -                    | +  |
| <i>less than three inches</i> | <i>less tall than</i>  | $\delta_1^r \triangleright_R \delta_2^r - 3$ |
| -                             | - +                    | +  |

## DIRECTIONAL ORDER AND DIRECTIONAL SUPREMUM SUPREMUM OF $\alpha$

If  $\alpha$  is a complex relation then the **directional order for  $\alpha$ ,  $>_\alpha$** , and the **directional supremum for  $>_\alpha$ ,  $\sqcup_\alpha$** , are defined by:

$$\begin{aligned} >_\alpha &= \begin{cases} >_H & \text{if the direction of } \alpha \text{ is } + \\ <_H & \text{if the direction of } \alpha \text{ is } - \end{cases} \\ \sqcup_\alpha &= \sqcup_{>_\alpha} \end{aligned}$$

### Examples:

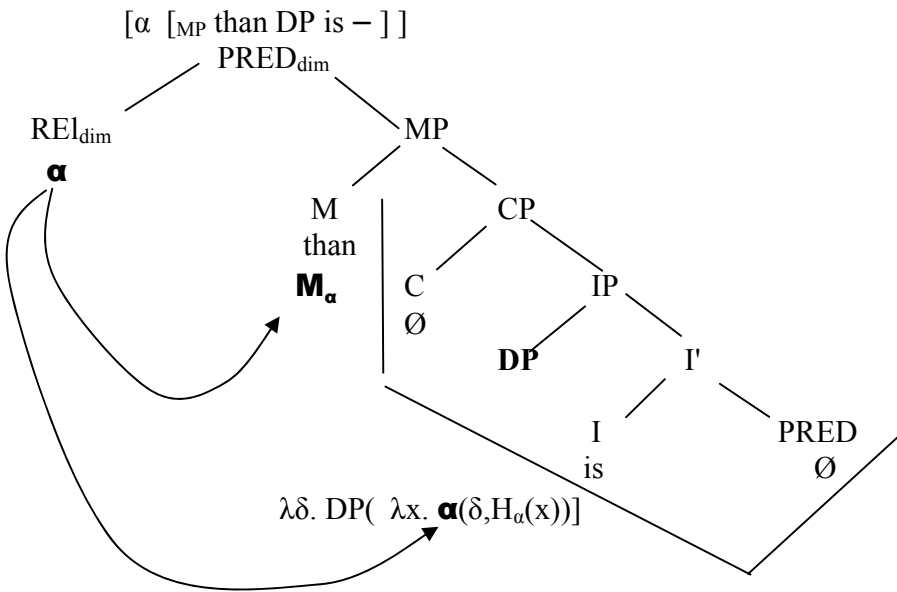
*more than three inches more tall than*

$$\alpha = \delta_1^f >_R \delta_2^f + 3 \quad \sqcup_\alpha = \sqcup_{>_H} \text{ (which corresponds to } \sqcap_{<_R} \text{)}$$

*less than three inches more tall than*

$$\alpha = \delta_1^f <_R \delta_2^f + 3 \quad \sqcup_\alpha = \sqcap_{>_H} \text{ (which corresponds to } \sqcup_{<_R} \text{)}$$

## THE DIRECTIONAL SUPREMUM THEORY OF CP COMPARATIVES



### -SWL:

-comparison relation  $\alpha$  is interpreted inside the comparative CP.

### -Directional supremum theory: comparison relation $\alpha$ has a **double** effect:

- comparison relation  $\alpha$  is interpreted inside the comparative CP.
- comparison relation  $\alpha$  determines the interpretation of the head  $M$  as operation  $M_\alpha$ .

**DIRECTIONAL SUPREMUM AS A PRESUPPOSITIONAL CHECK OPERATION  $M_\alpha$**

$$M_\alpha(\beta) = \begin{cases} \beta & \text{if } \sqcup_\alpha(\beta) \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

$M_\alpha$  is a **presuppositional** version of the identity function:

- $M_\alpha$  has no **semantic** effect on input set of degrees  $\beta$ ,  $M_\alpha(\beta)$  has **the same meaning** as  $\beta$ ,
- $M_\alpha$  **presupposes** that the dimensional supremum of  $\alpha$  of set of degrees  $\beta$  **exists**.

I assume here that  $\sqcup_{<R}$  and  $\sqcap_{<R}$  are **operations on  $R$** , and since  $\infty$  and  $-\infty$  are not in  $R$ , for our purposes here,  $\sqcap_{<R}(X)$  is **undefined** when  $X$  is left-unbounded, and  $\sqcup_{<R}(X)$  is undefined when  $X$  is right-unbounded.

$M_\alpha$ : like a definiteness operation, but without the type change from  $\langle d, t \rangle$  to  $d$ :

**$M_\alpha(\beta)$  presupposes that** the unique object  $\sqcup_\alpha(\beta)$  exists, even if it doesn't **denote** it.

**CONSEQUENCE:**

**If  $\beta_{CP}$  and  $\beta_{DP}$  are comparative correlates and the meaning of  $\beta_{CP}$  is defined, then  $\beta_{CP}$  and  $\beta_{DP}$  have the same meaning.**

1a) and (1b) are predicted to be equivalent, whenever (1b) is defined:

- (1) a. John is  $\alpha$  than DP
- b. John is  $\alpha$  than [ $_{CP}DP$  is – ]

**Thus: the dimensional supremum theory of CP comparatives is a presuppositional variant of SWL.**

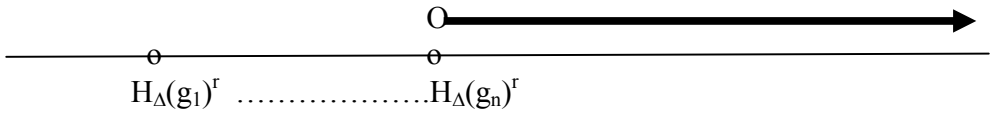
# DOWNWARD ENTAILING EXPRESSIONS INSIDE CP COMPARATIVES

(1) John is taller than *every girl* is –

taller:  $>_{\alpha} = >_H$   $\sqcup_{\alpha} = \sqcup_{>H}$  derived from  $\sqcup_{<R}$

CP: *than every girl is* –  $\beta = \lambda\delta.\forall y[GIRL(y) \rightarrow \delta^r > H_{\Delta}(y)^r]$

$\sqcup_{<}(\beta)^r$  defined **FELICITOUS**



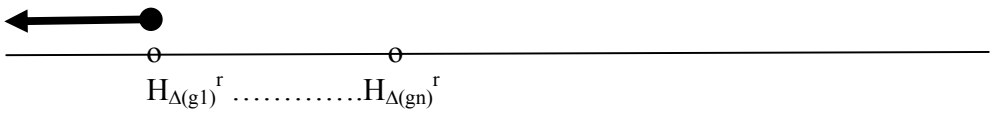
(2) #John is taller than *no girl* is –

taller:  $>_{\alpha} = >_H$   $\sqcup_{\alpha} = \sqcup_{>H}$  derived from  $\sqcup_{<R}$

CP: *than no girl is* –  $\beta = \lambda\delta.\neg\exists y[GIRL(y) \wedge \delta^r > H_{\Delta}(y)^r]$

$\sqcup_{<}(\beta)^r$  undefined

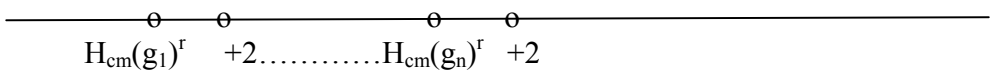
**INFELICITOUS**



(3) John is at most two cm taller than *every girl* is –.

at most two cm taller:  $>_{\alpha} = <_H$   $\sqcup_{\alpha} = \sqcup_{>H}$  derived from  $\sqcup_{<R}$

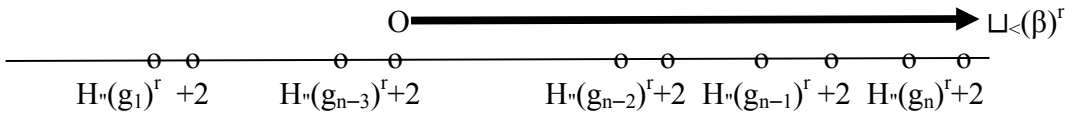
$\sqcup_{<}(\beta)^r$  defined **FELICITOUS**



(4) #John is at most two cm taller than *at most three girls* are –.

at most two cm taller:  $>_{\alpha} = <_H$   $\sqcup_{\alpha} = \sqcup_{>H}$  derived from  $\sqcup_{<R}$

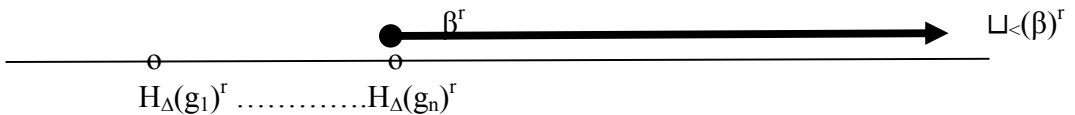
**INFELICITOUS**



(5) #John is less tall than *no girl* is –

less tall  $>_{\alpha} = <_H$   $\sqcup_{\alpha} = \sqcup_{>H}$  derived from  $\sqcup_{<R}$

**INFELICITOUS**



**FACT:** -Downward entailing noun phrases in the CP complements of comparatives are predicted to be infelicitous by the directional supremum operation.  
 -Non-downward entailing noun phrases are not predicted to be infelicitous by this mechanism.

## MODULAR SCALAR SEMANTICS AND RICH SCALES

-The CP-complement of the comparative: **set of degrees**

based on the external comparative relation: **relation between degrees**.

-The **scalar direction** of the external degree relation determines the **directional degree supremum**:

-depending on the scalar direction,  $\mathbf{M}_\alpha$  checks for a degree supremum **based on  $\sqcup_R$**  or **based in  $\sqcap_R$** .

Downward entailing expressions inside the CP-complement yield at the CP level a set of degrees which is unbounded on the side of the dimensional supremum:

-on the left side, in the case of  $+$ -relations (based on  $\sqcap_{<R}$ );

-on the right side in the case of  $-$ -relations (based on  $\sqcup_{<R}$ ).

This produces the infelicity.

### **Modularity of the semantics:**

The whole structure of directional relations, operations, their converses, and the corresponding directional supremum notions are most naturally formulated at the level of the scales (as properties of and relations between degrees).

-The structure of the scales as closed under basic arithmetics (allowing for subtraction to be defined), as closed under infimums and supremums, and as being unbounded in both directions has been exploited extensively.

-Note: the use of unboundedness in B requires a modular semantics, because, at the level of individual or mixed relations, the access of the scale is through the measure function, and hence only degrees in the **range** of the measure function are relevant.

But the range of the measure function is typically **not unbounded**:

e.g. the measure functions for Height and Loveliness have a null-point, and negative values are not assigned.

Hence: the analysis of CP-comparatives relies on **both** the modular scalar semantics **and** the rich scales assumed.

Turning matters around, the success of the analysis provides motivation for a modular scalar semantics and rich scales.

# LECTURE 5

## POLARITY ITEMS IN THE DIRECTIONAL SUPREMUM THEORY

- (1) **Mary is more famous than John *ever* was.**

**FAME:**  $F_{\Delta} : \text{individuals} \times \text{moments of time} \rightarrow \text{degrees of fame (measured in units of } \Delta \text{)}$ .  
 $[_{CP} \textit{John ever was } \neg_{famous}]$   
 $\lambda\delta. \exists t[\delta >_F F_{\Delta}(\text{John}, t)]$   
 The set of degrees  $\delta$  for which there is a time  $t$  such that  $\delta$  is bigger on the scale of fame than John's degree of fame at  $t$ .

We assume, with Kadmon and Landman 1993, that the semantic effect of *ever* is **widening**. Thus, you may have said (3A), and I reply (3B):

- (2) A. Mary is more famous than John is now.  
 B. Mary is more famous than John *ever* was.

### WIDENING IN CP COMPARATIVES

The widening is temporal, and involves a wide and a narrow interpretation:

- (3) a.  $\lambda\delta. \exists t[t \in \text{WIDE SET} \wedge \delta >_F F_{\Delta}(\text{John}, t)]$   
 b.  $\lambda\delta. \exists t[t \in \text{NARROW SET} \wedge \delta >_F F_{\Delta}(\text{John}, t)]$

Scalar comparison construction:

Standard assumption: widening is not unconstrained, but is along the scale.

**$\max_{\text{JOHN}, F}$**  is the **moment of time** where John's fame is maximal (for simplicity assume that there is one such time).

Pragmatic assumption: there must be a **point** to using *ever*, and hence to widening.

So: **Implicature:**  $\max_{\text{JOHN}, F} \notin \text{NARROW SET}$

Simplest assumption:

Widening done by *ever* just **adds**  $\max_{\text{JOHN}, F}$  to the narrow set:

**Widening:**  $\text{WIDE SET} = \text{NARROW SET} \cup \{ \max_{\text{JOHN}, F} \}$

Interpretation of the CP:

$[_{CP} \textit{John ever was } \neg_{famous}]$   
 $\lambda\delta. \exists t[t \in \text{NARROW SET} \cup \{ \max_{\text{JOHN}, F} \} \wedge \delta >_F F_{\Delta}(\text{John}, t)]$

By the assumptions made, John's degree of fame at  $\max_{\text{JOHN}, F}$  is higher than John's degree of fame at any of the points of time in the narrow set: so:

[<sub>CP</sub> *John ever was*  $\neg_{famous}$  ]  
 $\lambda\delta. \delta >_F F_{\Delta}(\text{John}, \mathbf{max}_{\text{JOHN}, F})$

Given these assumptions, in a natural context (1) is interpreted as:

**(1) Mary is more famous than John *ever* was.**

$F_{\Delta}(\text{Mary})^r >_F F_{\Delta}(\text{John}, \mathbf{max}_{\text{JOHN}, F})^r$

Mary's degree of fame(now) is higher than John's degree of fame when it was maximal.

This is an adequate account of the meaning of (1).

## THE LICENSING OF THE PS ITEM.

Kadmon and Landman: a PS item is licensed if **widening leads to strengthening** at the level of the closest relevant operator the PS item is in the scope of.

## STRENGTHENING IN CP COMPARATIVES

Main assumption: **strengthening is defined at the level of the comparative scale** (relation between elements of type  $\delta$ )

### SCALAR STRENGTHENING:

On scale  $S_M$ :  $\delta_1$  **strengthens**  $\delta_2$  iff  $\delta_1 \geq_M \delta_2$

On scale  $S_M^c$ :  $\delta_1$  **strengthens**  $\delta_2$  iff  $\delta_1 \leq_M \delta_2$

### DP comparatives:

-No grammatical level where the interpretation of the DP is of type  $\delta$ . (type  $d$  or  $\langle\langle d, t \rangle, t \rangle$ )

(Composition with the measure function takes place in the comparison relation, not in its object.) **Hence, scalar strengthening is irrelevant and PS items are not licensed in DP comparatives.**

### CP comparatives:

-No grammatical level where the interpretation of the CP is of type  $\delta$ .

-But the presuppositional check operation brings in an interpretation of type  $\delta$  as a presupposition:  $t_{\alpha}(\beta)$  of type  $\delta$ .

**Assumption: The polarity item is licensed if widening leads to strengthening at the presuppositional level  $t_{\alpha}(\beta)$**

This means that we check whether (2a) strengthens (2b):

- (2) a.  $\sqcup_{>F}(\lambda\delta.\delta >_F F_{\Delta}(j, \mathbf{max}_{\text{JOHN}, F}))$   
 b.  $\sqcup_{>F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta >_F F_{\Delta}(\text{John}, t)])$

$\sqcup_{>F}$  corresponds to  $\sqcap_{<R}$ , hence:

$$\sqcup_{>F}(\lambda\delta.\delta >_F F_{\Delta}(j, \mathbf{max}_{\text{JOHN}, F})) = F_{\Delta}(j, \mathbf{max}_{\text{JOHN}, F})$$

Let  $\mathbf{min}_{\text{narrow}, \text{JOHN}, F}$  be the time in NARROW where **John's fame is minimal in comparison to the other times in the narrow set**.

Obviously, the infimum of the degrees of John's fame corresponding to the times in the narrow set is  $F_{\Delta}(j, \mathbf{min}_{\text{narrow}, \text{JOHN}, F})$ .

$$\sqcup_{>F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta >_F F_{\Delta}(\text{John}, t)]) = F_{\Delta}(j, \mathbf{min}_{\text{narrow}, \text{JOHN}, F})$$

Thus, we are checking whether (3a) strengthens (3b):

- (3) a. **wide degree**  $F_{\Delta}(j, \mathbf{max}_{\text{JOHN}, F})$   
 b. **narrow degree**  $F_{\Delta}(j, \mathbf{min}_{\text{narrow}, \text{JOHN}, F})$

Clearly,  $F_{\Delta}(j, \mathbf{max}_{\text{JOHN}, F}) \geq_F F_{\Delta}(j, \mathbf{min}_{\text{narrow}, \text{JOHN}, F})$ , hence **(3a) strengthens (3b)**:

**CONSEQUENCE:**

**The polarity item *ever* is licensed in the CP comparative (1):**

- (1) **Mary is more famous than John *ever* was.**



We look at (4):

(4) **Mary is less famous than John *ever* was.**

-Widening involves  $\mathbf{min}_{\text{JOHN,F}}$ , the time where John's fame was minimal

The pragmatic assumption is that John's fame at  $\mathbf{min}_{\text{JOHN,F}}$  is smaller than all the times in the narrow set.

The supremums we get here are as in (5):

- (5) a.  $\sqcup_{<_F}(\lambda\delta.\delta <_F F_{\Delta}(\text{John},\mathbf{min}_{\text{JOHN,F}})) = \mathbf{min}_{\text{JOHN,F}}$   
b.  $\sqcup_{<_F}(\lambda\delta.\exists t[t \in \text{NARROW SET} \wedge \delta <_F F_{\Delta}(\text{John},t)]) = \mathbf{max}_{\text{narrow,JOHN,F}}$

The relevant dimension is the converse dimension, i.e., we are in  $S_F^c$ , so we use  $S_F^c$ -**strengthening** as defined above, and we see that indeed:

$$\mathbf{min}_{\text{JOHN,F}} \leq_F \mathbf{max}_{\text{narrow,JOHN,F}} \quad \text{(5a) strengthens (5b)}$$

**CONSEQUENCE:**

**The polarity item *ever* is licensed in the CP comparative (4):**

(4) **Mary is less famous than John *ever* was.**

## **MODULAR SCALAR SEMANTICS AND RICH SCALES**

-The CP-complement of the comparative: **set of degrees**

based on the external comparative relation: **relation between degrees.**

-The **scalar direction** of the external degree relation determines the **directional degree supremum**:

-depending on the scalar direction,  $M_{\alpha}$  checks for a degree supremum **based on  $\sqcup_R$**  or **based in  $\sqcap_R$ .**

Relevant for licencing of polarity items in CP-complements

**degree strengthening** for polarity items is measured **along the direction of the external comparative relation**, which is a modular scalar relation.

-Downward entailingness is not measured at an entailment scale but at a measure scale.

-Open question: why are free choice readings licensed in (DP and CP comparatives)?

# SUPREMUM DEGREE INTERPRETATIONS IN MODALS

## Heim 2006:

- (1) To be accepted into the police school, you have to be 1.65 and you can be 1.92.  
*have to be 1.65:* 1.65 is the **minimal height necessary** to be allowed in.  
*can be 1.92 in (1):* 1.92 is the **maximal height possible** to be allowed in.

## Modals inside comparatives:

- (2) a. Fortunately, John is taller than he has to be (to be let in).  
 = taller than then minimal height necessary  
 b. Unfortunately, Bill is taller than he can be (to be let in).  
 = taller than the maximal height possible

## Heim's analysis:

### Heim measure relation:

Given measure function  $M_{u,w}$ , the corresponding **Heim measure relation** is:

$M_{u,w}^*(x, \delta)$  iff  $0 < \delta^r \leq M_{u,w}(x)^r$

### Von Stechow comparative:

*taller than  $\varphi$*  –

$\lambda\delta. \delta >_H \sqcup_{<H} (\lambda\delta. \varphi(\lambda w \lambda x. H_{\Delta,w}^*(x, \delta)))$

**Classical modals:**  $\forall w \in ACC_{w0}, \exists x \in ACC_{w0}$

## Predictions:

- (3 $\forall$ ) John has to be 1.65 (to be let in)  
 $\forall w \in ACC_{w0}: H_{m,w}^*(John, 1.65)$   
 The minimal height required for John to be let in 1.65 **GOOD**
- (4 $\forall$ ) John is taller than he has to be (to be let in).  
 $H_{m,w0}(John) >_H \sqcup_{<H} (\lambda\delta. \forall w \in ACC_{w0}: H_{\Delta,w}^*(John, \delta))$   
 John is taller than the minimal height. **GOOD**
- (4 $\exists$ ) John is taller than he can be (to be let in)  
 $H_{m,w0}(John) >_H \sqcup_{<H} (\lambda\delta. \exists w \in ACC_{w0}: H_{\Delta,w}^*(John, \delta))$   
 John is taller than the maximal height. **GOOD**
- (3 $\exists$ ) John can be 1.95 (and be let in)  
 $\exists w \in ACC_{w0}: H_{m,w}^*(John, 1.95)$   
 In some world  $w \in ACC_{w0}$ :  $\lambda\delta. H_{m,w}^*(John, \delta) = (0, 1.95]$ .  
 No maximality interpretation imposed. **= PROBLEM ONE**

Disturbing asymmetry: Maximality readings for degrees in existential modals are as readily available as minimality readings are for degrees in universal modals.

## PROBLEM TWO

**right** predictions for the modals inside comparatives =

**wrong** prediction for explicit quantifiers inside comparatives:

(8<sub>∀</sub>) John is taller than every girl is -.

$$H_{m,w_0}(\text{John}) >_H \sqcup_{<H}(\lambda\delta. \forall y \in \text{GIRL}_{w_0}: H_{\Delta,w}^*(y,\delta))$$

John is taller than the smallest girl is -.

(8<sub>∃</sub>) John is taller than some girl is -.

$$H_{m,w_0}(\text{John}) >_H \sqcup_{<H}(\lambda\delta. \exists y \in \text{GIRL}_{w_0}: H_{\Delta,w}^*(y,\delta))$$

John is taller than the tallest girl is -.

## PROBLEM THREE

Heim restricts attention to contexts with a unique minimally acceptable height and a unique maximally acceptable height. Not the general case.

**Example:** minimally and maximally acceptable heights depend on a decision by the height-committee still to be made:

They will decide between 1.65 and 1.70 and between 1.90 and 1.95.

### Intuitively valid pattern:

- (13<sub>∀</sub>)
- This year, you have to be 1.65 or 1.70 to be accepted for the police school.
  - When I applied, I was taller than you have to be this year (and they didn't take me).
  - Hence, I was taller than **1.70**.

### Invalid pattern predicted by Heim:

- (13<sub>∀</sub>\*)
- This year, you have to be 1.65 or 1.70 to be accepted for the police school.
  - When I applied, I was taller than you have to be this year (and they didn't take me).
  - \*c. Hence, I was taller than **1.65**.

### Intuitively valid pattern:

- (13<sub>E</sub>)
- This year, you can be 1.90 or 1.95 and be accepted for the police school.
  - When I applied, I was shorter than you can be this year (and they didn't take me).
  - Hence, I was shorter than **1.90**.

### Invalid pattern predicted by Heim:

- (14<sub>∃</sub>\*)
- This year, you can be 1.90 or 1.95 and be accepted for the police school.
  - When I applied, I was shorter than you can be this year (and they didn't take me).
  - \*c. Hence, I was shorter than **1.95**.

### Diagnosis:

**These examples involve variation of minimal acceptable height and maximal acceptable height across the accessible worlds.**

## PROPOSAL Double modality:

The examples in (13):

1. **implicit modal** interpreted as a universal quantifier over accessible worlds
2. in (13) accessible worlds are worlds where the height John has to be varies.
3. in (14) accessible worlds are worlds where the height John can be varies.
4. The height John has to be in  $w$  is the **minimal acceptable height (for John) in  $w$**
5. The height John can be in  $w$  is the **maximal acceptable height (for John) in  $w$** .

(1)-(3) give the following semantics:

(17) a. John has to be 1.65 or 1.70 to be accepted for the police school.

$\forall w \in ACC_{w0}$ : **if**  $H_{m,w}(John) = \text{the height John has to be in } w$   
**then**  $H_{m,w}(John) = 1.65 \vee H_{m,w}(John) = 1.70$

(17) b. John can be 1.90 or 1.95 and be accepted for the police school.

$\forall w \in ACC_{w0}$ : **if**  $H_{m,w}(John) = \text{the height John can be in } w$   
**then**  $H_{m,w}(John) = 1.90 \vee H_{m,w}(John) = 1.95$

## Degree definites with modal restrictions:

**Explicit modal material** is part of an **implicit relative clause** on a definite degree head.

*the height John has to be – in  $w$*

*the height John can be – in  $w$*

1. Let  $\mathbf{acc}_w$  is the set of heights acceptable in  $w$  as John's height

Let  $\mathbf{acc}_w^b = \mathbf{acc}_w \cup \{\sqcap_{<H}(\mathbf{acc}_w), \sqcup_{<H}(\mathbf{acc}_w)\}$

2.  $\mathbf{ACC}_w$  is a set of worlds where John's height varies according to  $\mathbf{acc}_w$ :

We associate with  $\mathbf{acc}_w^b$  a set of accessible worlds  $\mathbf{ACC}_w$  with **constraint**:

For every  $\delta \in \mathbf{acc}_w^b$  there is a  $z \in \mathbf{ACC}_w$ :  $H_{m,z}(John) = \delta$

For every  $z \in \mathbf{ACC}_w$ : there is a  $\delta \in \mathbf{acc}_w^b$ :  $H_{m,z}(John) = \delta$

3. Degree definite:  $\sigma(\beta) = \sqcup_{<H}(\beta)$  if  $\sqcup_{<H}(\beta) \in \beta$ , undefined otherwise

## Degree relative:

*the height John has to be:*  $\sigma(\lambda\delta.\forall z \in \mathbf{ACC}_w: H(\delta, H_{m,z}(John)))$

*the height John can be:*  $\sigma(\lambda\delta.\exists z \in \mathbf{ACC}_w: H(\delta, H_{m,z}(John)))$

## Constraint on H

H is a relation between degrees such that:

$\sigma(\lambda\delta.\forall z \in \mathbf{ACC}_w: H(\delta, H_{m,z}(John))) = \sqcap_{<H}(\mathbf{acc}_w)$

$\sigma(\lambda\delta.\exists z \in \mathbf{ACC}_w: H(\delta, H_{m,z}(John))) = \sqcup_{<H}(\mathbf{acc}_w)$

-For example, Heim's relation works:  $H(\delta_1, \delta_2)$  iff  $0 <_H \delta_1 \leq_H \delta_2$

-Alternative: construct set of worlds  $ACC_w$  from set of degrees  $\mathbf{acc}_w$  through degree ideals:

**World construction:**

For  $\delta \in \mathbf{acc}_w^b$ :  $w_\delta = \{\delta' \in \mathbf{acc}_w^b : \delta' \leq_H \delta\}$ .

$ACC_w = \{w_\delta : \delta \in \mathbf{acc}_w^b\}$

*the height John has to be in  $w$  according to  $\mathbf{acc}_w$  to be accepted*

$\sigma(\lambda\delta.\forall z \in ACC_w : \delta \in z) = \sqcap_{<H}(\mathbf{acc}_w)$

*the height John can be in  $w$  according to  $\mathbf{acc}_w$  and be accepted*

$\sigma(\lambda\delta.\exists z \in ACC_w : \delta \in z) = \sqcup_{<H}(\mathbf{acc}_w)$

Valid inference:

(17) a. John has to be 1.65 or 1.70 to be accepted for the police school.

$\forall w \in ACC_{w0}$ : if  $H_{m,w}(\text{John}) = \sqcap_{<H}(\mathbf{acc}_w)$  then  $H_{m,w}(\text{John}) = 1.65 \vee H_{m,w}(\text{John}) = 1.70$

(17) b. I am taller than John has to be:

$\forall w \in ACC_{w0}$ : if  $H_{m,w}(\text{John}) = \sqcap_{<H}(\mathbf{acc}_w)$  then  $H_{m,w0}(\text{Fred}) >_H H_{m,w}(\text{John})$

(17) c. I am taller than 1.70

$H_{m,w0}(\text{Fred}) >_H 1.70$

Valid inference:

(18) a. John can be 1.90 or 1.95 and be accepted for the police school.

$\forall w \in ACC_{w0}$ : if  $H_{m,w}(\text{John}) = \sqcup_{<H}(\mathbf{acc}_w)$  then  $H_{m,w}(\text{John}) = 1.90 \vee H_{m,w}(\text{John}) = 1.95$

(18) b. I am shorter than John can be:

$\forall w \in ACC_{w0}$ : if  $H_{m,w}(\text{John}) = \sqcup_{<H}(\mathbf{acc}_w)$  then  $H_{m,w0}(\text{Fred}) <_H H_{m,w}(\text{John})$

(18) c. I am shorter than 1.90

$H_{m,w0}(\text{Fred}) <_H 1.90$

**Advantage 1.** The asymmetry between the universal modal cases and the existential modal cases is removed. (i.e. problem one is resolved)

**Advantage 2.** The analysis no longer relies on the analysis of the comparatives: we get the correct results with the almost (but not quite) naïve theory of comparatives, without relying on von Stechow's otherwise problematic theory (i.e. problem two is resolved).

**Advantage 3: the analysis is tailored to the more general case:**

The variation of the minimal/maximal possible heights in degree modals is analyzed as variation of the definites *the height John has to be* and *the height John can be* across the worlds accessible according to an implicit accessibility relation (i.e. problem three is resolved).

**Advantage 4:** The Heim minimality/maximality effects are moved from the degree phrases to the interpretation of **degree abstraction inside modal relatives**.

Work on degree relatives (e.g. Carlson 1977, Heim 1987, Grosu and Landman 1998, Landman 2000, Grosu and Krifka 2007, Landman ms.) has precisely argued that the internal semantics of degree relatives involves internal operations like maximalization.

This is the natural place for modal minimalizer/maximalizer operations like H to be introduced in the derivation.

The analysis puts the modal minimalizer/maximalizer effects in the grammatically plausible place.